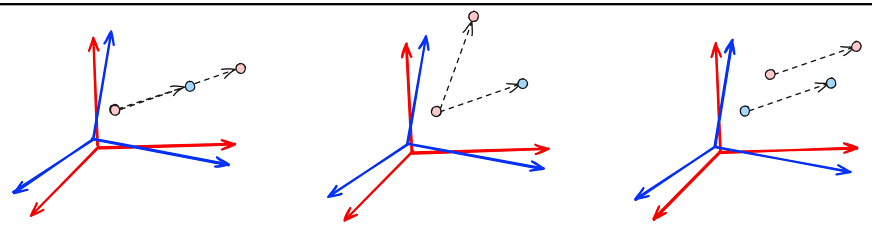

CE331 Lab : Coordinate Transformation

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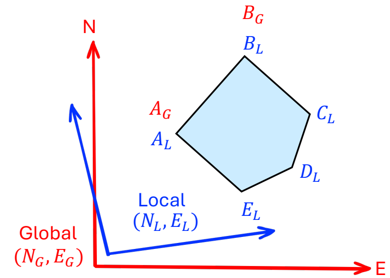
Coordinate Transformation

2D Cartesian Coordinate Transformations

- Changing point positions from one coordinate system to another

$(N_L, E_L) \longrightarrow (N_G, E_G)$

Local Coordinate System → **Global Coordinate System**



Transformation Elements

Scaling

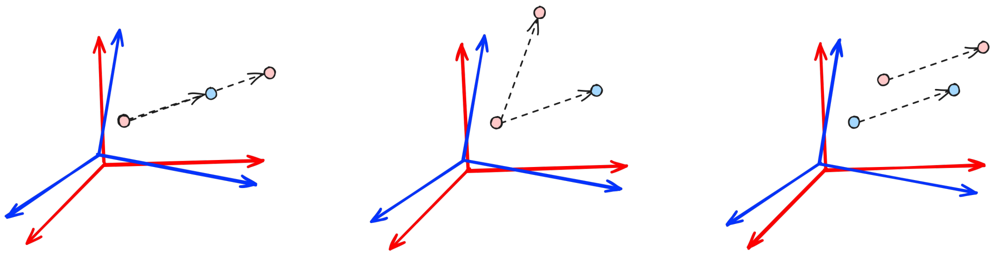
- Resizes data points by multiplying coordinates by a constant factor

Rotation

- Rotates data points around a specified origin by a certain angle

Translation

- Moves data points by adding a constant offset to their coordinates



Affine Transformation Equations

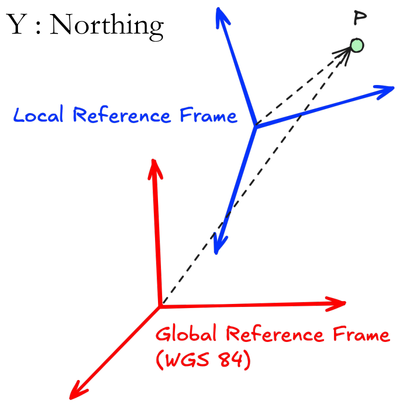
$$X_G = S \cdot X_L \cos(\rho) - S \cdot Y_L \sin(\rho) + T_x$$

$$Y_G = S \cdot X_L \sin(\rho) + S \cdot Y_L \cos(\rho) + T_y$$

X : Easting

Y : Northing

- *Local Coordinates :* (X_L, Y_L)
- *Global Coordinates :* (X_G, Y_G)
- T_x, T_y *: Translation in X and Y direction*
- S *: Uniform scale factor (both direction)*
- ρ *: Rotation angle (in radians)*



Matrix Form of Affine Transformation

- The affine transformation can be represented in matrix form as:

$$\begin{pmatrix} Y_L & X_L & 1 & 0 \\ X_L & -Y_L & 0 & 1 \end{pmatrix} \begin{pmatrix} S\cos(\rho) \\ S\sin(\rho) \\ T_y \\ T_x \end{pmatrix} = \begin{pmatrix} Y_G \\ X_G \end{pmatrix}$$

$$AX = L$$

$$A = \begin{pmatrix} Y_L & X_L & 1 & 0 \\ X_L & -Y_L & 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} S\cos(\rho) \\ S\sin(\rho) \\ T_y \\ T_x \end{pmatrix} \quad L = \begin{pmatrix} Y_G \\ X_G \end{pmatrix}$$

Solution

- Number of unknowns in X = 4
- Number of equations from one point = 2
- At least two points are required to get the solution
- For redundant scenario, use least squares solution

Procedure:

- **Step 1:** Use at least two known points to estimate unknown parameters (X).

$$X = (A^T A)^{-1} A^T L$$

- **Step 2:** Apply the affine transformation to calculate global coordinates of all control points.

$$AX = L$$

Procedure

Local Coordinates

(N_{l1}, E_{l1})
(N_{l2}, E_{l2})
.
.
(N_{ln}, E_{ln})

Known

Global Coordinates

(N_{g1}, E_{g1})
(N_{g2}, E_{g2})

Known

.
.
(N_{gn}, E_{gn})

unknown

STEP 1 Put local and global coordinates of CP1 and CP2 in AX=L to find the solution.

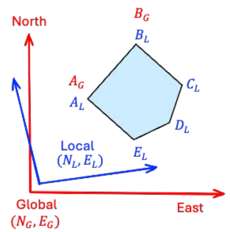
$A_1 X = L_1$
$A_2 X = L_2$

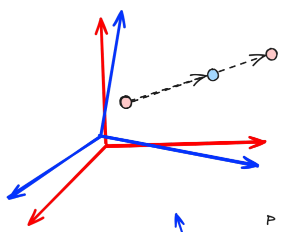
Solve them to find X= ?
$X = (A^T A)^{-1} A^T L$

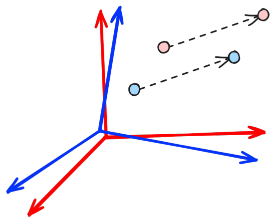
STEP 2 Use computed X to find global coordinates of all the control points.

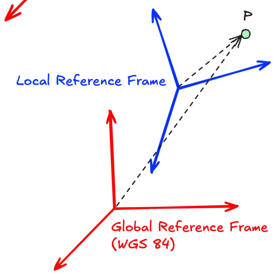
$L_3 = A_3 X$
$L_4 = A_4 X$
. .
. .
$L_n = A_n X$

global coordinates of remaining control points









Comments and Questions?

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