COORDINATE TRANSFORMATION

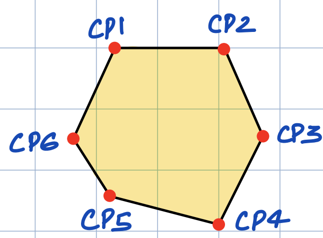
Given local coordinates of 'n' control points and global coordinates of 2 control points, we want to find global coordinates of other remaining 'n-2' control points.

AFFINE TRANSFORMATION EQUATION

$$\boxed{\begin{aligned} N_G &= S \cdot \sin(\rho) \cdot N_L + S \cos(\rho) \cdot E_L + T_N \\ E_G &= S \cdot \cos(\rho) \cdot N_L - S \cdot \sin(\rho) \cdot E_L + T_E \end{aligned}}$$

Representing above equation in matrix form as $\boxed{AX = L}$

$$\underbrace{\begin{bmatrix} N_L & E_L & 1 & 0 \\ E_L & -N_L & 0 & 1 \end{bmatrix}}_{\text{Transformation matrix } (A)} \underbrace{\begin{bmatrix} S \cdot \cos\rho \\ S \cdot \sin\rho \\ T_N \\ T_E \end{bmatrix}}_{\text{unknowns } (x)} = \underbrace{\begin{bmatrix} N_G \\ E_G \end{bmatrix}}_{\text{observations } (L)}$$



$N_G, E_G \rightarrow$ GLOBAL COORDINATES

$N_L, E_L \rightarrow$ LOCAL COORDINATES

$T_N, T_E \rightarrow$ TRANSLATION IN NORTH / EAST

$\rho \rightarrow$ ROTATION ANGLE

$S \rightarrow$ SCALE FACTOR (ASSUMED SAME FOR NORTH/EAST)

No. of unknowns $(x) = 4$

No. of equations from one point $= 2$

Therefore, we need at least 2 points to get the solution.

The solution is given by $\boxed{X = (A^T A)^{-1} A^T L}$

PROCEDURE

Assuming we know global coordinates of CP1 and CP2.

	Local Coordinates	Global Coordinates
CP1	(N_{l1}, E_{l1})	(N_{g1}, E_{g1})
CP2	(N_{l2}, E_{l2})	(N_{g2}, E_{g2})
⋮	⋮	⋮
CPn	(N_{ln}, E_{ln})	(N_{gn}, E_{gn})

Local coordinates of CP1 … CPn: known

Global coordinates of CP1, CP2: known

Global coordinates of CP3 … CPn: unknown

STEP 1 Put local and global coordinates of CP1 and CP2 in $AX = L$ to find the solution.

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \qquad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

Solve them to find $X = ?$

$$X = (A^T A)^{-1} A^T L$$

For redundant solution (>2 points), least squares is used.

STEP 2 Use computed X to find global coordinates of all the remaining CPs.

$$\left.\begin{aligned} L_3 &= A_3 X \\ L_4 &= A_4 X \\ &\vdots \\ L_n &= A_n X \end{aligned}\right\}$$

global coordinates of remaining control points.