CE331A

GEOINFORMATICS

Prof. Balaji Devaraju

Aman

L1

27/09/2022

Prof. Balaji Devaraju
expertise.
Satellite Gravimetry
Signal Processing
Research
Env. Geodesy

metrology (science of measurement)

surveying (position, plan) → metrology

↓

Map → setting out

↓

work

↓

surveying → Map

Relative Measurements → Ref. Frame (coordinate)

- Distances → size
- Angles → Shape / Orientation
- Heights → Topography.
  Topographic Maps.

* Measurements → errors
  Adjustment Theory

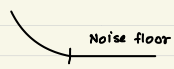
* Precision → How many measurements you can take
  Accuracy → How close your measurement is to true value

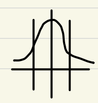
Principle of surveying (Holy grail of surveying)

Whole to part
↳ constraint
- minimal no. of observations → optimal
- highly precise

- This gives me constraint to my parts to sit within the whole
- less errors that we make.

$$\bar{x} = \sum_{i=1}^{N} \frac{x_i}{N}$$





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* Redundancy (extra checks)
  → Tests blunders
  error : systematic effect

* For a good measurement

$$\frac{\text{signal power}}{\text{noise power}} > 2$$

* 10 cm ± 5 μm → pecision of the output
  0.2 mm

↓

It determines the no. of observations that we need to take

1 : X (scale) & size of the whole
(Logistics) optimal no. of observations.

"Geoinformatics Consultant" → Hot word

Tomorrow ⤳ About Control surveying
How to do it? (About whole).

L2

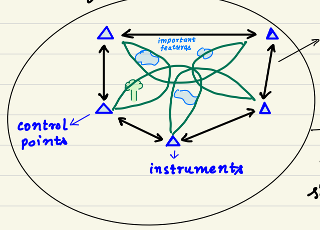
28/09/2022

Control Networks

Whole to part | Distances, Angle, Height | firm ground, intervisibility, optimal no. of points

Control Survey

establishing the network for the whole

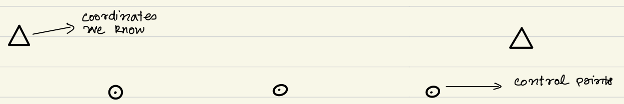


- Look for firm ground
- Intervisibility
- Optimal no. of points

} Reconnaisance

Step 1: Reconnaisance
Step 2: Establishing Control
Coordinates of the control points

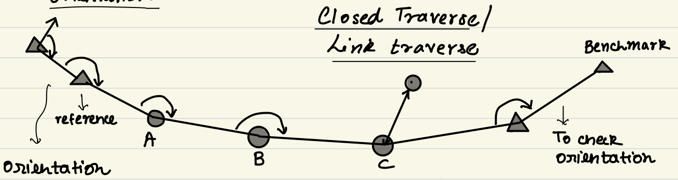
Traverse → lowest order of control survey
Triangulation } Highly precise varieties → precision
Trilateration }



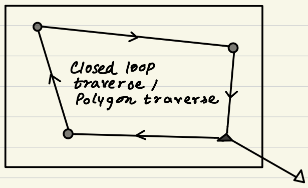
$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

All our measurements are inherently deficient in
→ reference information → Datum Problem
→ orientation

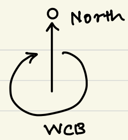
Closed Traverse / Link traverse



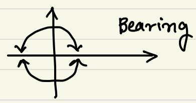
Simple Traversing : Establish points and add control point as a reference point. We add another control point to determine the orientation of our and then to check for that our orientation is right, we add two or more control points at the end of our traversal.

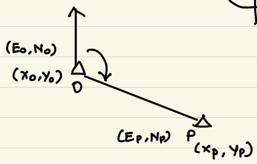


Planar Survey → We do not take curvature of earth into consideration.
→ Planar coordinates
Eastings | Cartesian
Northings |
Polar coordinates
Distances &
Whole Circle Bearings



Clockwise angle from North (0° to 360°) is WCB.

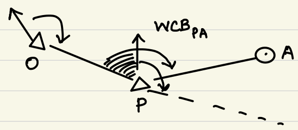




$d = \sqrt{(E_0 - E_P)^2 + (N_0 - N_P)^2}$

$WCB_{OP} = \alpha = \arctan\left(\frac{E_P - E_0}{N_P - N_0}\right)$

WCB_{OP} ⤳ talking about the orientation



$WCB_{OP} + \angle OPA$

sum = $WCB_{OP} + \angle OPA$

If sum > 180° | −180
sum < 180° | +180
sum > 360°
} ⤳ Doubtful ??

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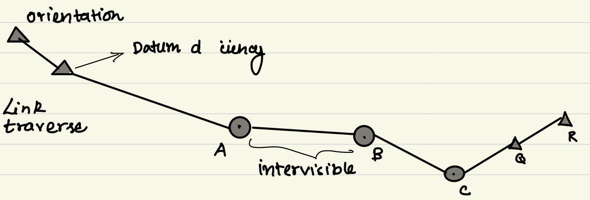
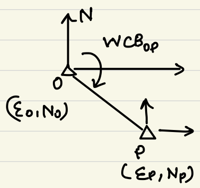


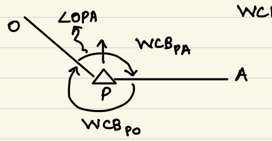


$d = \sqrt{(E_P - E_0)^2 + (N_P - N_0)^2}$

$WCB_{OP} = \tan^{-1}\left(\frac{E_P - E_0}{N_P - N_0}\right)$

$WCB_{PA} = WCB_{OP} + \angle OPA + 180°$

$WCB_{PO} = 180° + WCB_{OP}$



WKT $WCB_{PO} + \angle OPA > 360°$

$WCB_{PO} + \angle OPA - 360°$

$WCB_{OP} + 180 + \angle OPA - 360°$

$WCB_{OP} + \angle OPA - 180°$

$E_A = E_P + d_{PA} \sin WCB_{PA}$

$WCB_{AB} = WCB_{AP} + \angle PAB$

$WCB_{BC} = WCB_{BA} + \angle ABC$

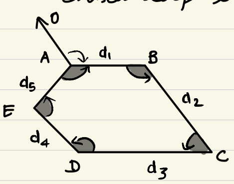
$WCB_{CQ} = WCB_{CB} + \angle BCQ$

$WCB_{PA} = WCB_{PO} + \angle OPA$

L4

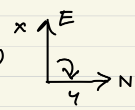
18/10/2022

Control Surveys — Traversing
Closed loop traverse



distances } Coordinates
angles

Whole circle bearing → (CW)
↳ angle ✗
↳ orientation ✓



$d\cos(WCB) = \Delta N$ } North as y-direction [Latitude]
$d\sin(WCB) = \Delta E$ } East as x-direction [Departure]

(E_A, N_A) → Known
$(\tilde{E}_A, \tilde{N}_A)$ → Computed
Known with uncertainty

random — unavoidable
systematic — estimatable
blunders — avoidable

Adjustment :- Bowditch (Hand-calculation)
(Rigorous) Least square adjustment

$$E_A + \begin{bmatrix} \Delta E_{AB} \\ \Delta E_{BC} \\ \Delta E_{CD} \\ \Delta E_{DE} \\ \Delta E_{EA} \end{bmatrix} \quad N_A \begin{bmatrix} \Delta N_{AB} \\ \Delta N_{BC} \\ \Delta N_{CD} \\ \Delta N_{DE} \\ \Delta N_{EA} \end{bmatrix} = (E_B, N_B)$$

$\sum = 0$ 0

Acceptable Errors

Internal $(n-2)\,180°$
External $(n+2)\,180°$

$\delta\alpha$ → Error

$\sigma_\omega^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2 + \sigma_E^2 = 5\sigma^2$

$\sigma_\omega = \sqrt{5}\,\sigma$

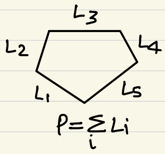
$-3\sigma_\omega < \delta\alpha < 3\sigma_\omega$

Schofield Exercise 6·1

Bowditch

$\delta E = \sum_i \Delta E_i \qquad \varepsilon_i = \frac{\delta E}{P} \times L_i$

$\delta N = \sum_i \Delta N_i \qquad \eta_i = \frac{\delta N}{P} L_i$



$P = \sum_i L_i$

Observation Condition | Cost function

$\min \left[\sum (y - Ax)^2 \right]$

$y = (A)x$
mapping
mathematical model / Design

Larger scale → more accurate maps

L5

19 / 10 / 2022

Control surveys : Traverse and Triangulation

Traverse procedure :

- Reconnaissance
- Control point identification
- Distance and Angle measurement
- Adjustment → within my tolerance → bowditch → Least squares
- Coordinates

Relative Measures

$\sum_i \Delta E_i = \delta E$

$\sum_i \Delta N_i = \delta N$

$e = \sqrt{\delta E^2 + \delta N^2}$

Relative error $= e/p$ → perimeter of the traverse

$= \frac{1}{(P/e)}$

1/100

10 cm → 10 km

$\frac{10 \times 10^{-2}}{10 \times 10^{3}}$

1/100,000

Book Schofield Ex. 6·1

Control Network → Tolerance

density ↓

Primary	1/10,000,000	→ 1/1000000
Secondary	1/100,000	
Tertiary	1/10,000	

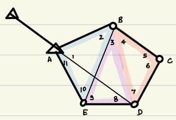
[Lower order feature mapping]

1/100 Large — 1/100000 small

1mm

Triangulation

Traverse

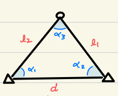


more redundant conditions

Triangle — 1 condition

Quadrilateral — >1 condition

m − observations
n − parameters } (m − n) condition equations



$l_1 = \sqrt{(x_c - x_B)^2 + (y_c - y_B)^2}$

$l_2 = \sqrt{(x_c - x_A)^2 + (y_c - y_A)^2}$

$\frac{\sin\alpha_1}{l_1} = \frac{\sin\alpha_2}{l_2} = \frac{\sin\alpha_3}{d}$ → observed

pseudo observations $\begin{cases} l_1 = \frac{\sin\alpha_1}{\sin\alpha_3} d \\ l_2 = \frac{\sin\alpha_2}{\sin\alpha_3} d \end{cases}$

< 25 km → Planar

> 25 km → Spherical / Ellipsoidal

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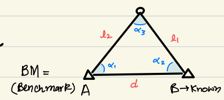
Control Surveys − Triangulation



$l_1 = \frac{\sin\alpha_1}{\sin\alpha_3} d$
$l_2 = \frac{\sin\alpha_2}{\sin\alpha_3} d$ } pseudo observations

↳ intermediate step

Observations —non-linear→ parameters → uncertainties
linearized
↳ dissolve the parameters from other quantities

$y = Ax$
$y = f(x)$

Initial Value

$y = f(x)\big|_{x = x_0} + \frac{\partial}{\partial n} d$ } Taylor Expansion

$\alpha_1 + \alpha_2 + \alpha_3 = 180°$

α_i = sum / difference of whole circle bearings

$WCB = \arctan\left(\frac{X_P - X_Q}{Y_P - Y_Q}\right)$

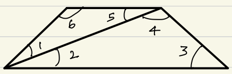
↳ Linearise it then get initial values from pseudo observations

3 useful observations $\begin{cases} \alpha_1 = \\ \alpha_2 = \\ \alpha_3 = \end{cases}$

$d = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$ → Abnew information

redundancy = 3 − 2 = 1

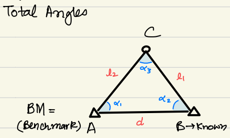
Condition eq^n :– $\alpha_1 + \alpha_2 + \alpha_3 = 180°$
3 angle
{ 2 unknown WCB
1 condition eq^n



condition eq^n = 6 − 4 = 2 (No. of lines of unknown WCB; Total Angles)

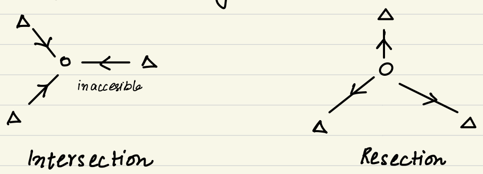
$l_1 = \sqrt{(X_B - X_C)^2 + (Y_B - Y_C)^2}$

$\cos\alpha_1 = \dfrac{l_2^2 + l_3^2 - l_1^2}{2 l_2 l_3}$



{Triangulation + trilateration = Triangulateration}
⟶ Primary Network

Primary ⟶ Secondary



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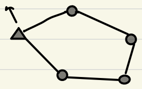
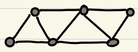
25/10/2022

Coordinate Systems — Reference Surfaces

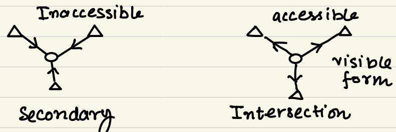
Traversing



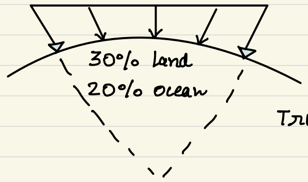
Triangulation,
Trilateration,
Triangulateration



Resection



Network of control points that have coordinates



True shape of Earth → Geoid
(Greek word)
Earth-like

Equopotential surface
↓
Gravity potential = Gravitation + centrifugal acceleration
(Gravitation: due to mass; centrifugal acceleration: due to rotation)

Shape of earth is determined by density composition.
Geoid is equopotential surface that is closest to the mean sea level (MSL).

Centering → in line with target
Levelling → due to gravity
we align ourselves with the plumb line.

Shape of the Earth } → Geoid : gravity + geometry

Mapping on Earth } → Coordinates → No gravity information is required; only geometry

Geoid :→ Best fitting regular surface → Ellipsoid
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In case of geoid, we take an ellipse and rotate it along an axis.

Equation : $\dfrac{x^2+y^2}{a^2} + \dfrac{z^2}{c^2} = 1$

Ellipsoid :–
Two types of ellipsoid
1. Physical ellipsoid → has mass, rotation, equopotential surface
2. Geometric ellipsoid

Geoid		Ellipsoid
mass	→	mass – density is uniform
rotation	→	rotation
flattening	→	flattening

↳ $f = \dfrac{a-b}{a} \Rightarrow \dfrac{1}{f} \approx 300$

Plane ——— Surveying & Mapping {Ch-8 Schofield}
Planar coordinates : (E,N,U) RHS
(N,E,U) LHS

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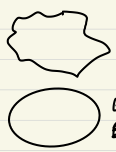
26/10/2022

Coordinate Systems – Reference Surfaces

— Plane ⟶ Local



True shape
⟶ Closest to the MSL
70% of the earth

Global / Regional — $\bar{h}$ = few metres

$\bar{h} = \dfrac{1}{4\pi} \int_{\Omega} h(\phi,\lambda)\cos\phi \, d\phi \, d\lambda \to 0$

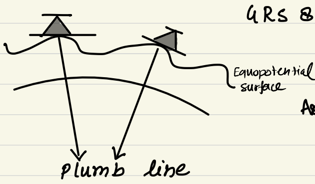
Geodetic Model of the Earth
Ellipsoid → Geometric Model
→ Mass } Physical model
→ Rotation }
→ flattening $f = \dfrac{a-b}{a}$ ⟹ $1/f \approx 300$

SRTM ⟶ ETOPO
(Shuttle Radar Topography Mission)

→ equopotential surface

gravity acceleration = gravitation (mass) + centrifugal acceleration (rotation)

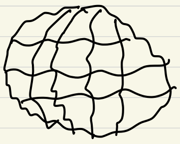
Level Ellipsoid :– Ellipsoid which also have gravitational field.
WGS 84
GRS 80



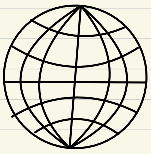
Enclosed topology
ϕ, λ → Polar coordinates (ϕ latitude, λ longitude)

Astronomical survey { Φ – Astronomical Latitude
Λ – Astronomical Longitude }

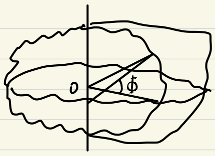
Geodetic Astronomy + Spherical Trigonometry

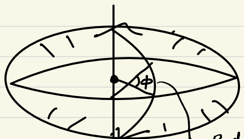


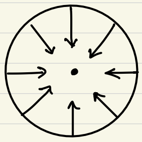
Mapping X → Potential of the gravity field of earth
Φ, Λ, W
RHS

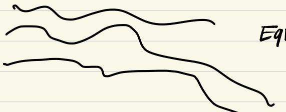


Geodetic Coordinates
ϕ, λ, h → Ellipsoidal height
Latitude, Longitude
RHS









Equipotential surfaces are not parallel

(Φ, Λ, W) → physical → geopotential → Vertical coordinate → Datum
geometric

Horizontal Coordinate (ϕ, λ, h) → Ellipsoidal height (3D coordinate height system)
↳ Datum → Reference System

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27/10/2022

Coordinate Systems — Transformations

Natural Coordinate system — (Φ, Λ, W) (Astro Lat., Astro Long., Gravity Potential)

Geodetic coordinate — (ϕ, λ, h) (Geodetic Lat., Geod. Long., Ellipsoid height, Ellipsoid ref.)

Ellipsoid Coordinates — (u, β)

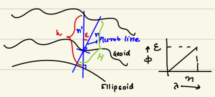
$W \rightarrow h$

$C = W_0 - W$ (potential on the geoid)

$H = \frac{C}{g}$ $\frac{m^2/s^2}{m/s^2}$ (physical quantity; geometric quantity; m)

→ orthometric height

gravity = ∇ geopotential



(Φ, Λ, H)
(ϕ, λ, h)

xi $\xi = \Phi - \phi$
eta $\eta = (\Lambda - \lambda)\cos\phi$
} Deflections of the vertical

Gradient of the geoid

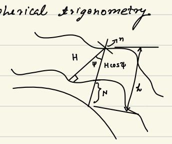
This factor takes care of the convergence of lat. & long.

psi $\psi = -(\xi \cos\alpha + \eta \sin\alpha)$ → We don't use this in surveying.

azimuth in spherical trigonometry

$h = N + H\cos\psi$ (geoid undulation; ortho height; Ellipsoid Height)

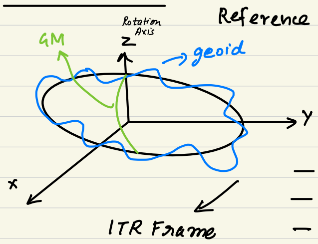
$\cos 60'' =$
$h \approx N + H$ cm (mm, mm)



$\xi = \Phi - \phi$ (→ GNSS) } gradient of the geoid
$\eta = (\Lambda - \lambda)\cos\phi$
$N = h - H$ → undulation of the geoid

(ϕ, λ, h) — 3D Coordinate System

Transformations — Datum
Reference Frame



International Terrestrial Reference System (ITRS)

- orientation of x, y, z
- origin
- orientation in space
- Geophysical Phenomena

Stable coordinates + Ellipsoid + Origin + orientation
— Geophysical activity)

Chapter 8 Schofield Book

L10

29/10/2022

Coordinate System — Transformations

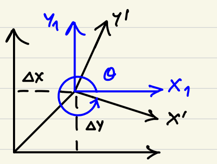
Astronomical/Natural Coordinate System (Φ, Λ, H) (from geopotential W; orthometric height)

Geodetic Coordinate System (ϕ, λ, h) (ellipsoidal height)

Coordinates on the geoid.
$\Phi = \phi - \xi$
$\Lambda = \lambda - \eta\sec\phi$
$H = h - N$ (geoid undulation)

Two maps → different coordinate systems
↓
reconciliation
↓
Coordinate Transformations

Datum
ob solute



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R(\theta)\begin{bmatrix} x \\ y \end{bmatrix} + R(\theta)\begin{bmatrix} \Delta y \\ \Delta x \end{bmatrix} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad x_1 = x + \Delta x,\ y_1 = y + \Delta y$$

$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} x' \\ y' \end{bmatrix}$

Case 1 : translation known in (x, y) system

- translation of the origin
- Rotation of the axes

① = ②

Rotation Matrices are orthonormal

skew-symmetric

translation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \underset{LHS}{P}\, R_x(\gamma)\, R_y(\phi)\, R_z(\lambda) \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

(Lattitudinal Difference; Longitudinal; → RHS)

$x = (\gamma + h)\cos\phi\cos\lambda$

$y = (\gamma + h)\cos\phi\sin\lambda$

$z = f(\gamma)\sin\phi$

γ – Radius of curvature of the prime vertical.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ $\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$\lambda = |\vec{r_1} - \vec{r_2}|$

$\Delta\vec{r}_{12}$

$R(\alpha,\beta,\gamma)\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} - \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right)$

$|R(\alpha,\beta,\gamma)\Delta\vec{r}_{12}|$

$\Delta\vec{r}_{12}\, R^T R\, \Delta\vec{r}_{12}$

Scale

- 3 translations
- 3 rotations
- 1 scale

⟹ 7 parameter similarity (→ Affine) Transformation

→ We use ϕ transformation in Image Processing !!

7 parameters Δx, Δy, Δz — mm to m

α, β, γ — $< 60''$

1 — 0.9992 to 1.01

L11

2/11/2022

Coordinates – Map Projections

3D Geodetic Coordinates —Projections→ Maps → 2D Medium

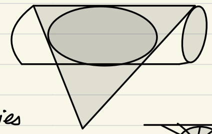
↓ Curvature of the geodetic model ellipsoid

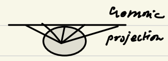
3D → 2D with distortion → Area, Angles, lengths (← directions, bearing)

minimizing the distortion

3D → 2D ellipsoid

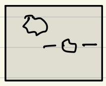
↓ 3D object — with distortion / without distortion → 2D

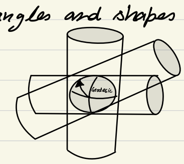




Objects	Distortion properties
Lambert Azimuthal	equal area projection
Lambert Conic	conformal → preserve angles and shapes

Mercator





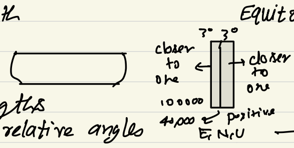
Great Circle ⟹ Passing through center of sphere

Geodesic → shortest distance passing through the points

Compromises

- Natural Earth
- Robinson
- Polyconic
- preserve lengths, shapes and relative angles

Equitorial Radius ≃ 111.11 km



$6° = 600.62$

— UTM

Field survey → map

L12

3/11/2022

Coordinate Systems – Projections & Height Systems

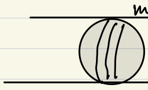
UTM → useful for regions that are NS elongated

60 zones

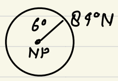
84° N – 80°S

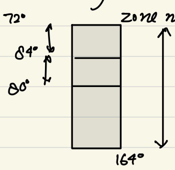
Poles – Polar Stereographic Projection

minimal distortion



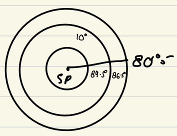
it preserves shapes orthomorphism → Shape/form

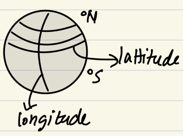




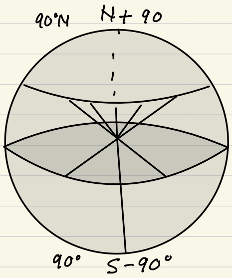
72°–84°N

164° – 20 tiles





Latitude is not greater circle except equator.



(ϕ, λ, h) — Geodetic (angles; distances) — Polar Coordinate

(X, Y, Z) — Cartesian (linear)

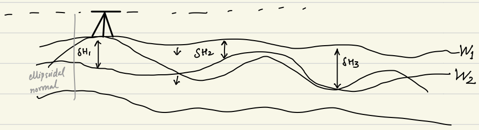
Heights

→ Unit of gravity

1 Gal = 1 cm/s²

m Gal = 10^{-3} cm/s²

0.01 m Gal = 10^{-5} cm/s²



$W_2 - W_1 = \delta W$

$\delta H_1 \neq \delta H_2 \neq \delta H_3$

$\delta W = \boxed{g}\, \Delta H$ (gravity → Plumbline)

Gravity $= \nabla W$ (↳ mass + rotation)

$g = \frac{\partial W}{\partial H} \Rightarrow \Delta W = g\, \Delta H$

$\delta W = g_1 \delta H_1 = g_2 \delta H_2 = g_3 \delta H_3$

$\Rightarrow H = \frac{W_0 - W}{\bar{g}}$ (Potential; force; orthometric Height)

$\bar{g} = \frac{1}{M} \int_{geoid}^{topography} g\, dH$

$\bar{g} = g_{topo}$ + Prey reduction

othometric height $H = \frac{W_0 - W}{\bar{g}}$

Dynamic height (ships) $= \frac{W_0 - W}{\gamma_0}$ $\left(\frac{m^2/s^2}{m/s^2}\right)$

Normal Height $H^* = \frac{W_0 - W}{\bar{\gamma}}$

$\bar{\gamma} = \frac{1}{H^*} \int \gamma\, dH^*$

L13

9/11/2022

Satellite Positioning – GNSS (Space-based surveys)

1. Laborious nature of surveying
2. Speed of surveying
3. Access to Benchmarks
4. Line of sight to control pts.

$10^{-3} \to$ milli
$10^{-6} \to$ micro
$10^{-9} \to$ nano
$10^{-13} \to$ pico

Benchmarks → Flying
→ High precision } Position
→ High accuracy

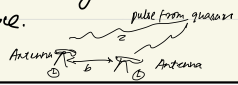
– Position of the flying benchmark
– Communicating the Position

Measurements of flying objects :-
Velocity
Angles / Directions (Azimuth, Elevation)
Doppler measurements → Good receiver → Δf (mm, sub mm)
– Range, range rate (send pulse, it reflects from satellite) (Atomic Clock)
– Time of arrival → Time is demanding
↳ Most efficient and cost effective.

① TRANSIT Positioning System → GPS — USA
GLONASS — Russia
Galileo — Europe
Bei Dou — China
IRNSS- NavIC — India
Q ZSS — Japan
} ToA, 2 Clocks, Every day positioning

Doppler → DORIS – French
Doppler – orbitography
Range, range rate – Satellite Laser Ranging
Very Long Baseline Interferometry (VLBI)
} Specialized – Geodetic, Astronomical

Quasars – Objects in space.

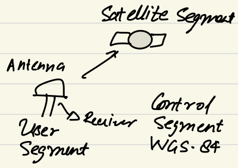


Global Navigation Satellite System
– ToA → fundamental principle
– Atomic Clocks → Critical technology → 4 atomic
– Microwave freq. → penetrate cloud cover → communication
↳ L-band   1-2 GHz   ($1-2 \times 10^9$ Hz)
$\lambda f = c$
$\lambda = \frac{c}{f} = 5\text{-}30$ cm

$f = 1.5 \pm 0.2$

L_1	E_1	B_1
L_2	E_2	B_2
L_5	E_5	B_5

Segments of a GNSS
– Satellite
– Control
– User



Ephemeris → position info of satellite

Key Parameters
– Position → Ephemeris
– Time tag
} → modulated over a carrier wave ↳ L-band

(Division Multiple Axis)
Code DMA – CDMA → Different Code
Frequency DMA – FDMA → Different Frequency.

Carrier → Messenger
Information → Positioning
Code → Speaker

Need to view 4 satellites
4 unknowns = 3 position + 1 clock error

(x_s, y_s, z_s) time of sending
$R = c\tau$
$\rho = c\tau +$ errors
pseudo range
time of receipt (x_R, y_R, z_R)

Observation Model:

Navigation Solution
$$\begin{cases} \rho_1 = \sqrt{(X_1 - X_R)^2 + (Y_1 - Y_R)^2 + (Z_1 - Z_R)^2} + c\delta t_1 \\ \vdots \\ \rho_4 = \sqrt{(X_4 - X_R)^2 + (Y_4 - Y_R)^2 + (Z_4 - Z_R)^2} + c\delta t_4 \end{cases}$$

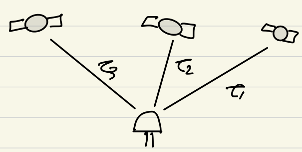
L14

9/11/2022

Satellite Positioning – GNSS

Benchmark – fly
Doris – DORIS
Laser ranging – SLR
time of arrival – GNSS, VLBI
} Too complex and require heavy instrumentation } Establishing reference frames.
surveying

GNSS → Global Navigation Satellite Systems
Principle – Resection



$\rho_i = R_i + c\delta t_i$

precision is diff. $T_{sat} - t_{receiver}$ low precision
$R_i = c\tau_i$
$\vec{X}_{receiver}$, $\vec{X}_{satellite}$ (known)
3 → unknowns.
1 → time

$\rho_i = R_i + c\,\delta t_i$ → Basic Observation Equation

$= (\vec{X}_{ir} - \vec{X}_i^s) + c\delta t_i$ geometry of the network ↓ Satellites that are in the purview of the receiver

linearised
$$\rho_i - \rho_{i_0} = \begin{bmatrix} \frac{\partial R_i}{\partial X_{ir}} & c \end{bmatrix} \begin{bmatrix} \delta X_r \\ \delta Y_r \\ \delta Z_r \\ \delta t \end{bmatrix}$$
Design

$l = A x$

residual range → l; design matrix → A; parameters → x

$$\hat{x} = \underbrace{(\underset{n\times m}{A^T}\underset{m\times n}{A})^{-1}\underset{n\times m}{A^T}}\ \underset{m\times 1}{l}$$

operator that maps the observation → parameters

$\hat{x} = N^{-1} d$, where $N = (A^T A)$ and $d = A^T l$

$A_{m\times n} \to$ Rank $= n$ (smaller of m and n)
we have less columns

$$Q_{\hat{x}\hat{x}\ (n\times n)} = (A^T A)^{-1} = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ | & \sigma_{yy}^2 & \sigma_{yz} & \sigma_{yt} \\ | & & \sigma_{zz}^2 & \sigma_{zt}^2 \\ | & - & - & \sigma_{tt}^2 \end{bmatrix}$$

$GDOP = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_{tt}^2}$

Dilution of Precision

PDOP =

$HDOP = R\, Q_{xx} R^{-1}$

$VDOP = \sqrt{\sigma_E^2 + \sigma_N^2}$

$Q_{\hat{x}\hat{x}} = \underbrace{(A^T A)^{-1}}_{N^{-1}} \sigma_u^2$ — trace (N^{-1}) = scale factor; m = accuracy

time precision $= \frac{1m}{c} = 3.33\times 10^{-3}$ sec.

$\sigma^2 = \frac{V^T V}{r}$ → posterior varience factor

$V = l - A\hat{x}$

$r = m - n$

$\rho_i = R_i + c\delta t_i$ → Basic Observation Eq^n

$\rho_i = R_i + c\delta t_i + T + I +$ Multipath + Instrumentation errors.

T → Topospheric refraction → 30 – 40 cm

I → Ionospheric refraction → 10 cm – few m

L15

10/11/2022

Satellite Positioning — GNSS

$\rho \to |\vec{X}_r - \vec{X}_s| \to S_\rho = \rho - \rho_0 = A\,\delta x$ (A: design Jacobian)

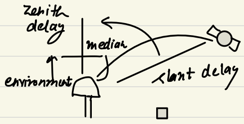
all the observations to be uniformly weighted.

GDOP $= \text{trace}[(A^T A)^{-1}]$ → A multiplication factor to the observation std. dev.

P
H
V
T

Geometry of the satellite network. ↪ Resection

$\rho = \hat{\rho} + c(\delta t_r - \delta t^s) + T + I$
+ receiver delay + Multipath
+ Antenna plane centre → you do not need mm precise measurement. do not care of if



Troposphere = CO_2, N, O_2, water vapour, feeling of weather, other greenhouse gases, inert gases.

Hydrostatic — modelled; Wet — estimated

Vienna Mapping Function

Ionosphere → modelling — Klobuchar model — carrier code data

frequency of the carrier wave

(The higher the frequency, the stronger the delay)

L-band
L_1, L_2, L_5 | ionosphere free combination

Combination of the observables at the different frequencies → eliminate ionosphere delay.

Ionosphere state

Total electron content = TEC

Surveying — (X, Y, Z) → precise

Tropo, Iono } spatial behaviour