Triangulation & Trilateration

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Aman

Triangulation & Trilateration

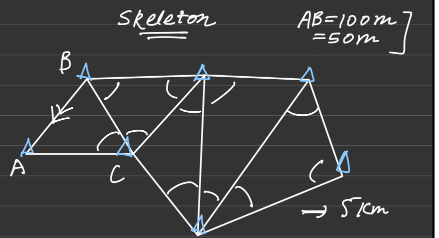
1. Why Triangulation, Example of use of triangulation, Classical Triangulation method, Base measurement, Different orders of triangulations, Figures, Criteria to select a figure, Great Trigonometric Survey of India.
2. Shape of a triangle, Strength of a figure, Field work in triangulation, Signals used in triangulation. Limitations, Satellite Station, Resection and Intersection.
3. Trilateration, Field work, Coordinate computation and adjustment Triangulateration.

Triangulation : Δ^n

↳ What? How?

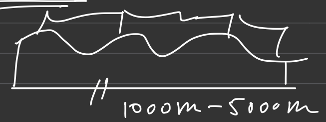
- Basically it is a network of triangles which we are trying to do in order to establish a control network.

Base length



$\theta \Rightarrow$ Thedolite

Simple Cases: Invar tape in Catenary

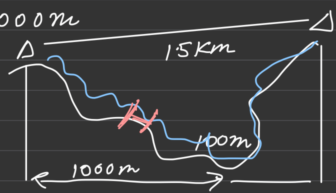


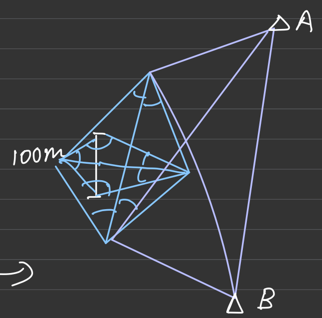
Principle of Triangulation

- In triangulation, whole area is covered with a framework of triangles.
- Principle:- If length and dir^n of one side and all the three angles of a Δ are measured precisely then length and dir^n of remaining two sides can be determined.
- The precisely measured first line is called Base-Line.
- Furthermore the other two lines whose length and direction are now known, act as base line for the other interconnected triangles. This process goes on further which gives rise to a network of triangles throughout the area to be surveyed.

- Check-Base: When the whole area is covered with triangles, then at last, as a check, the length of one side of the sides of the last triangle is also measured directly and compared with the computed value. This side is called Check Base.

Base length is 2 km 5 km



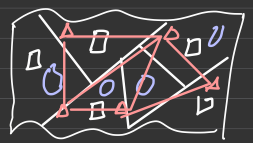


Base Extension :- ⟶

Why Δ^n :→

Establishing the Control Network
↳ Horizontal
↳ Vertical

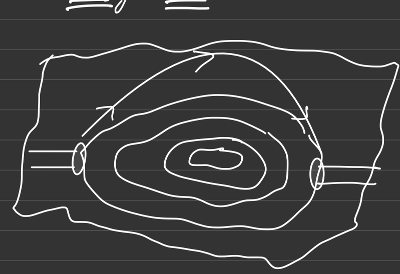
Whole to part

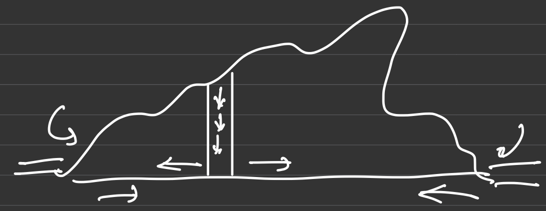


Laplace Station :- Station where we are taking the bearing.

Check Base

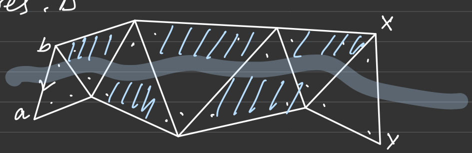
Setting out





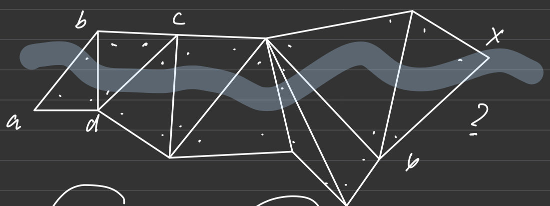
Figures : Δ^n

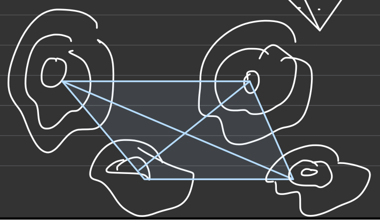
(1)



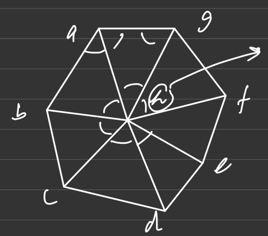
Route for Computation

(2) Braced Quadrilaterals



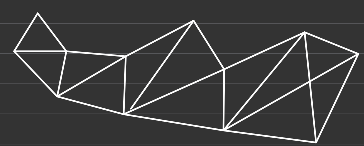


(3) Centered Figure

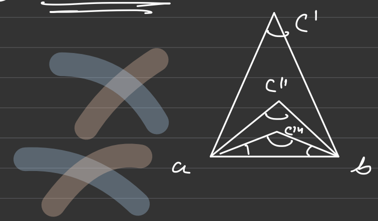


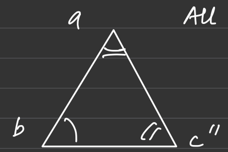
→ we have to occupy this 'h' also here.

How to select a figure?



(1) Computation from two routes
(2) well conditioned routes





All the angles of this triangle $30° < \theta < 120°$ ⇒ our triangle is **well conditioned**

→ computation of unknown pt. are accurate more.

(3) Cover the entire area.

Frame Work of a Country

↳ Grid Iron.

Centered Figure: Entire country (small) can be covered by triangle, then it is a centered figure.

Order of Triangulation

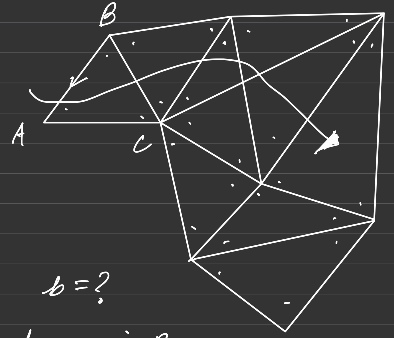
Characteristics	First order (Primary)	Second order (Secondary)	Third order (Tertiary)
Length of base	8 to 12 km	2 to 5 km	0.1 to 0.5 km
Length of sides	18 to 150 km	10 to 25 km	2 to 10 km
Average Triangle Closure	1''	3''	12''

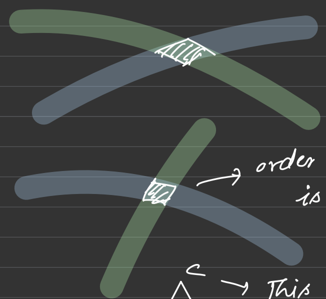
Great Trigonometric Survey

- Begun in April 1802 by Colonel William Lambton
- George Everest
- Many others....
- It helped to lay out a reference system for India.

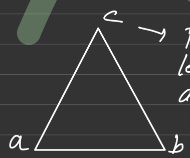
Shape of a Δ

(2)





order of uncertainty is less if $\angle = 90°$



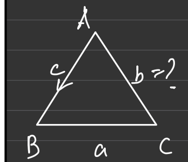
This 'c' will be located with good accuracy if this angle is 90°

↓

If very large ∠ or very small ∠ then error will be more.

$b = ?$

$$b = \frac{c \sin B}{\sin C}$$



σ_B, σ_C → standard errors

↳ $= \sigma_\theta$ (for some weather condition)

$$\sigma_b^2 = \left\{\left[\frac{\partial}{\partial B}\left(\frac{c \sin B}{\sin C}\right)\right]^2 \sigma_B^2\right\} + \left\{\left[\frac{\partial}{\partial B}\left(\frac{c \cdot \sin B}{\sin C}\right)\right]^2 \sigma_C^2\right\}$$

$$\boxed{\sigma_b^2 = b^2 \cdot \cot^2 B \cdot \sigma_B^2 + b^2 \cdot \cot^2 C \cdot \sigma_C^2}$$

same as σ_θ^2

$$\sigma_b^2 = b^2 \sigma_\theta^2 [\cot^2 B + \cot^2 C]$$

$$\frac{\sigma_b}{b} = \sigma_\theta [\cot^2 B + \cot^2 C]^{1/2}$$

min^m const

↳ to make it minimum ⇒ $\{B \& C ⇒ 90°\}$ × not possible otherway $\{A \text{ and } C ⇒ 90°\}$ ×

Ideal situation $A = B = C = 60°$ Not practical.

- If the angles are between (30° − 120°) then ↳ **well conditioned**.

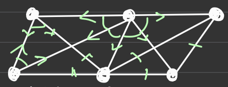
Strength of Figure

$$L^2 = \frac{4}{3} d^2 \, \frac{D-C}{D} \left\{ (\delta A^2 + \delta A \delta B + \delta B^2) \right\}$$

Probable error in computation in a chain of Δs.

probable error in angle measurement 60

D = total no. of dirⁿs observed except along known lines.



$$C = (n' - s' + 1) + (n - 2s + 3)$$

n' ⇒ no. of lines observed in both dirⁿ ; $D = 11 \times 2 - 2 = 20$

n ⇒ no. of lines

s' = stations occupied.

s = no. of stations

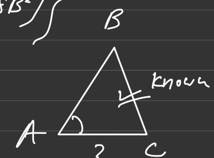
$$C = (11 - 6 + 1) + (11 - 2 \times 6 + 3) = 8$$

Using these we can determine $\frac{D-C}{D}$, which will be const. for a figure.

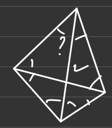
$$L^2 = \frac{4}{3} d^2 R \quad \text{if} \quad R = \frac{D-C}{D}\left\{\Sigma(\delta A^2 + \delta A \delta B + \delta B^2)\right\}$$

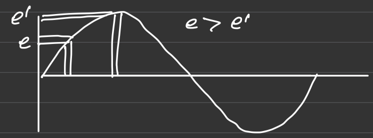
Strength of figure $\propto \frac{1}{R}$

δA = difference per second in the 6th place of log of sin of 'A'.



Distant Angle



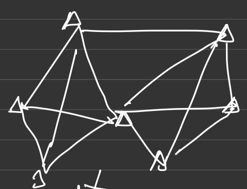


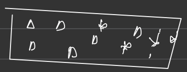
In all diff figures $\Sigma(\delta A^2 + \delta A \delta B + \delta B^2)$

this value is least will be best in computation.

Field Work in Δⁿ:

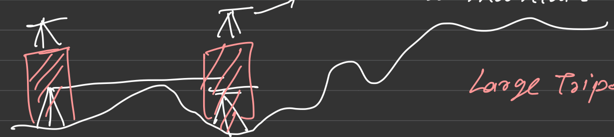
- Reconnaissance
- Measurement of Base line & Check Base
- Measure all the angles
- Angle Adjustment.
- Length of lines and Bearings.
- Computing coordinates.
- Plot them in sheet.



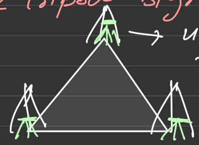


Signals and Towers

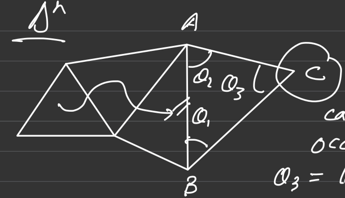
Towers to maintain intervisibility.



Large Tripod Signal



under tripod, total station is setup. if required.



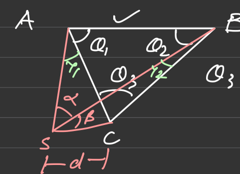
cannot be occupied

$\theta_3 = 180 - (\theta_1 + \theta_2)$

↳ errors can go unnoticed!

$\Sigma\theta \neq 180°$

Satellite Station: False Eccentric



$\theta_3 = ?$

$\theta_3 = 180 - (\theta_1 - \theta_2)$ → 1st approx. value

ΔABC AB

AC and BC

ΔASC

$SC = d$

$AC =$

$\angle ASC = \alpha + \beta$; $\angle SAC = \gamma_1$

ΔSBC $\angle SBC = \gamma_2$

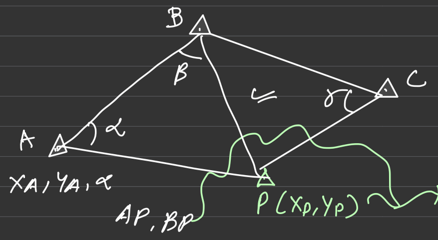
$$\angle BHC = 180 - (\alpha + \gamma_1)$$

$$\theta_3 = 180 - ((180 - (\alpha + \gamma_1)) + \gamma_2)$$

$$= \alpha + \gamma_1 - \gamma_2$$

$$\boxed{\theta_3 = \alpha + \gamma_1 - \gamma_2}$$

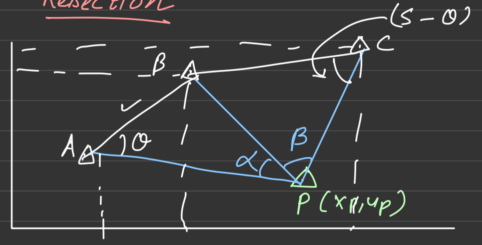
Intersection



locating a point by intersecting lines from two known landmarks on the ground.

→ you can't occupy this point

Resection



ΔABP

$BP = AP \cdot \dfrac{\sin\theta}{\sin\alpha}$ (sine rule)

ΔBCP

$BP = BC \dfrac{\sin(S-\theta)}{\sin\beta}$ (sine rule)

$\theta \rightarrow$ having known it, we can compute the coordinates (X_P, Y_P)

Why we did Triangulation?
we just need to measure a single length and all angles. so all the coordinates can be calculated. as in earlier days when EDMI was not there measuring lengths was difficult. Measuring angles is easier.

Trilateration :-

Now EDMI is there, we can easily compute all the lengths.

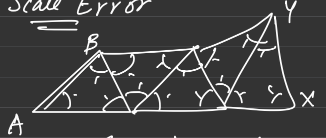
↓ compute all the lengths and establish a control network of triangles like earlier.

↓ It was thought it can be a better method of establishing control

Advantages of Trilateration

① Rapid :- straight away you are measuring distances
② It can control Scale Error

Scale Error



Error in angles → this error propogates → compute XY will have error bcoz of error in AB as well as how my angles are. also the size of angles

XY, error AB

→ But in Trilateration, we are computing these length, we are measuring them as such so there is no scale error.

Disadvantages of Trilateration

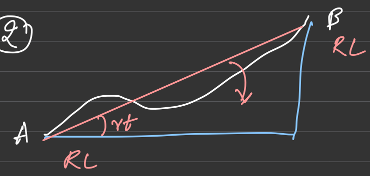
①

Internal angle conditions are not there as in case of triangulation.

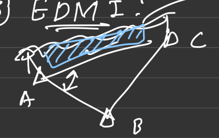
$F(l) = ?$
↓ l_1, l_2, l_3
There is no such condition in triangulation.

$\Sigma\theta \neq 180°$ → There is a way to know if error is there in triangulation.

②



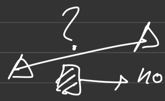
③ EDMI : → Atmospheric conditions are also a factor that we feed in the EDMI.



They may not be accurate.

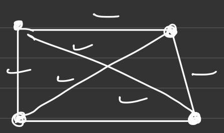
Field Work in Trilateration

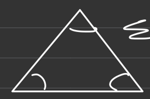
(1) Reconnaissance:
- ↳ Observe
- ↳ Stations



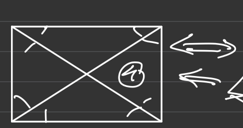
→ no industry otherwise atmospheric condition will be different.

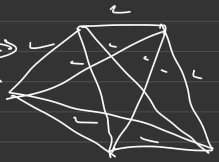
Figures:



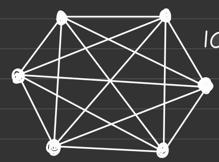


$\Sigma\theta = 180°$





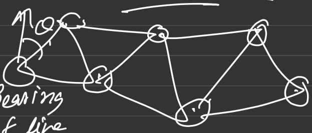
Researchers have suggested that the best figure in trilateration is hexagon with all the sides measured.



10 Conditions
20 Observ.

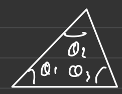
→ Length

Field Work



Bearing of line

Computation and Adjustment



$\Sigma\theta \neq 180°$

θ_1 systematic error

θ_2

θ_3

θ_1'

θ_2'

θ_3'

$\Sigma\theta' \neq 180°$

Random Errors

Adjustments

→ By least square we adjust all the system as a whole.

(1) Angle of adjustment

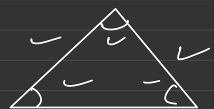
(2) Bearing of all lines

Trilateration:

→ Angles Adjustment.

Triangulation
Trilateration
Triangulateration

Triangulateration



Total Station → can measure angle and lengths simultaneously.
θ, l

All angles and all lengths measured → Very very precise control network.