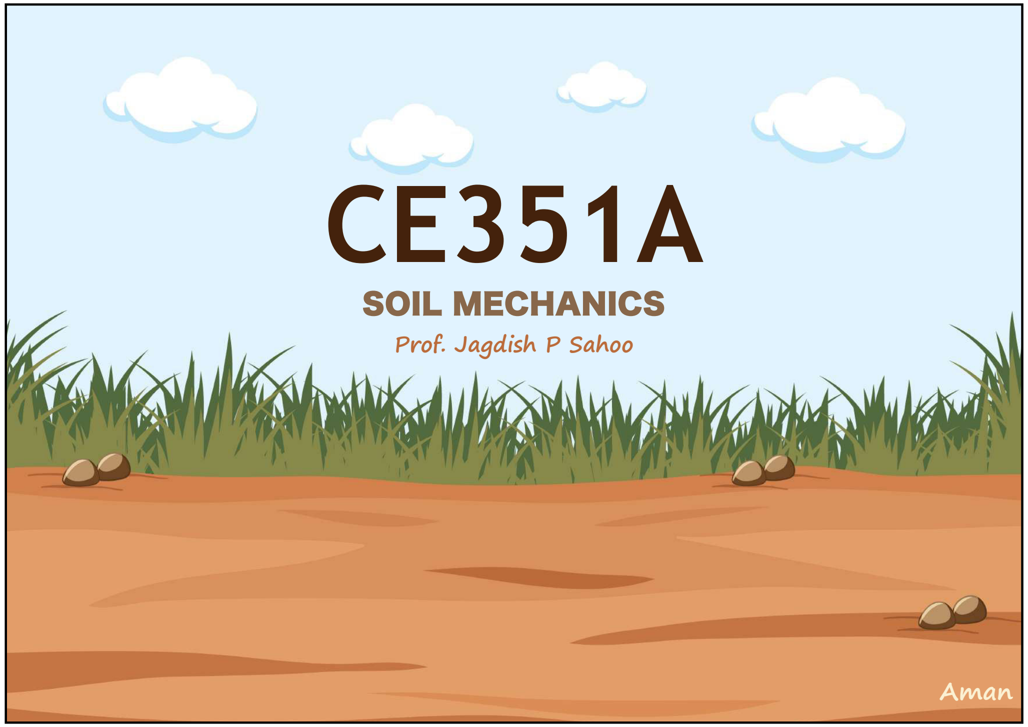
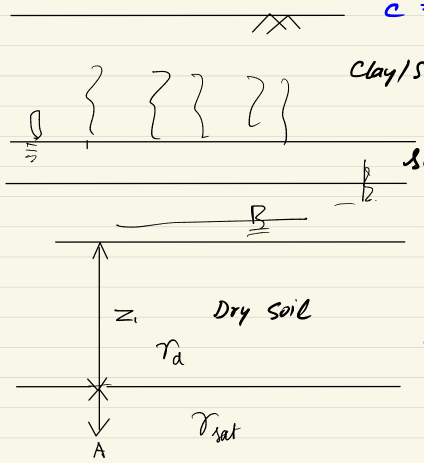


Empirical Correlation

$d = e^{1/3} D_{10}$

$h_c = \dfrac{c}{e D_{10}}$

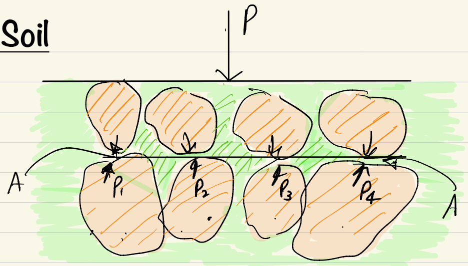
e = void ratio
D_{10} = effective size of the particles
d = equivalent diameter of the capillary tube
h_c = height of the capillary rise in the tube
c = constant = $10\,mm^2 - 50\,mm^2$ (size of the particles)



Case II : $\sigma_A = \gamma_d z_1 + \gamma_{sat} z_2$

Case I : $\sigma_A = \gamma_w z_3 + \gamma_{sat}(z_1 + z_2)$

Effective Stress Concept of Saturated Soil (Terzaghi, 1943)

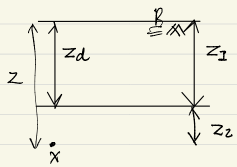


P_s = sum of vertical forces at the point of centre b/w the particles
$= P_{sv_1} + P_{sv_2} + P_{sv_3} + \dots + P_{sv_n}$

$P = P_s + P_w = P_s + u_w A_w$

where A_w = c/s area of water along the plane AA
u_w = pore water pressure
A = c/s area of the plane AA
(1 to 3%)A = A_s = sum of c/s area at the point of contact b/w particles

$\dfrac{P}{A} = \dfrac{P_s}{A} + \dfrac{u_w A_w}{A}$ $(A_w \cong A)$



$\sigma = \dfrac{P_s}{A} + u_w$

$\sigma = \sigma' + u_w$

$\sigma = \gamma_{sat} z$

$\sigma_x' = \gamma_{sat} z - \gamma_w z = \gamma' z$

$\sigma_x' = \gamma_d z_1 + z_2 \gamma'$

σ' = effective stress
↳ This is not a physical parameter

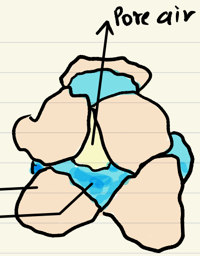
Effective Stress Concept of Unsaturated Soil (Bishop et. 1960)

$\sigma = \sigma' - u_a + K(u_a - u_w)$

$P = P_s + P_a + P_w$
$= P_s + u_a A_a + u_w A_w$
$= P_s + u_a(A - A_w)$

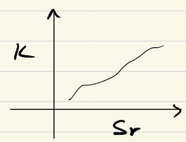
$\dfrac{P}{A} = \dfrac{P_s}{A} + u_a - \dfrac{u_a A_w}{A} + \dfrac{u_w A_w}{A}$

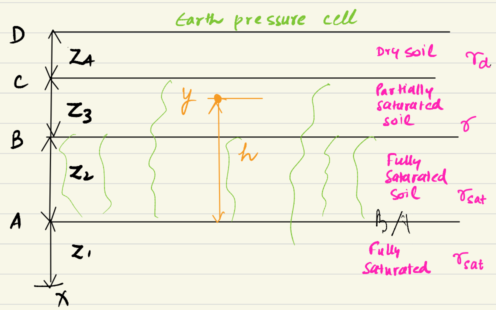
$\sigma = \sigma' - u_a + \dfrac{A_w}{A}(u_a - u_w)$



Soil suction

Tensiometer





$\sigma_D = 0$

$\sigma_C = \gamma_d z_4$ $S_r < 100\%$

$\sigma_B = \gamma_d z_4 + z_3 \gamma$ $S_r = 100\%$

$\sigma_A = \gamma_d z_4 + z_3 \gamma + z_2 \gamma_{sat}$

$\sigma_x = \gamma_d z_4 + z_3 \gamma + (z_2 + z_1)\gamma_{sat}$

Total

$\sigma_D = 0$

$\sigma_C = \gamma_d z_4$

$\sigma_B = \gamma_d z_4 + \gamma z_3$

$\sigma_A = \gamma_d z_4 + \gamma z_3 + \gamma_{sat} z_2$

$\sigma_x = \gamma_d z_4 + \gamma z_3 + \gamma_{sat} z_2 + \gamma_{sat} z_1$

Pore Water

$u_D = 0$

σ_C just above $= 0$

σ_C just below $= u_w'$

$u_w' = -\frac{s}{100} h \gamma_w$

$= \frac{s}{100}(z_2 + z_3)\gamma_w$

s = Degree of saturation

h = height of the location under consideration from the ground water table

Neutral Stress Pore Pressure

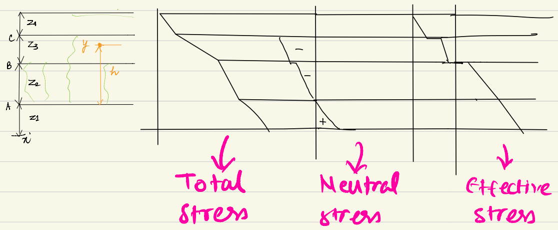
$u_D = 0$

u_C above $= 0$

u_C below $= -\frac{s}{100}(z_2 + z_1)\gamma_w$

u_B above $= -\frac{s}{100} z_2 \gamma_w$

u_B below $= -z_2 \gamma_w$



Effective Stress

$\sigma_D' = \sigma_D - u_D = 0$

σ_C' above $= \gamma_d z_d$

σ_C' below $= \gamma_d z_d + \frac{s}{100}(z_2 + z_3)\gamma_w$

σ_B' below $= \gamma_d z_4 + z_3 \gamma + z_2 \gamma_w$

$\sigma_A' = \gamma_d z_d + z_3 \gamma + z_2 \gamma_{sat}$

$\sigma_x' = \gamma_d z_d + z_3 \gamma + (z_2 + z_1)\gamma_{sat} - z_1 \gamma_w$

Flow through Soil Mass

* permeability is the ability that allows the flow of any liquid / fluid through it.

Clay → very low to low → can be considered impermeable / impervious soil

Silt → low to medium

Sand → high

Gravel → very high

* Laminar flow

Turbulent flow → $Re < 2000$ (Pipe flow)

$Re \leq 75$ (soil)

$Re \leq 1$

$Re = \frac{\rho v D}{\mu}$

ρ – Density of fluid

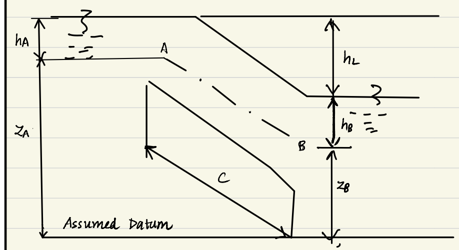
v – velocity of fluid

D – Average size of the particle

μ – Dynamic viscosity $\left(\frac{Ns}{m^2}\right)$

Henry Darcy

Hydraulic head



v = velocity of flow $\propto \frac{h_L}{L}$

$v \propto i$ (hydraulic gradient)

$v \propto \frac{h_L}{L}$

Total head = Datum head (Elevation head) + Pressure head (Piezometric head) + Velocity head

$= z + \gamma_w h + \frac{v^2}{2g}$ → Neglected because velocity of flow is very small in soil / mass

Total head at A $= H_A = z_A + h_A$

Total head at B $= H_B = z_B + h_B$

$h_L = z_A - z_B + h_A - h_B$

Darcy's Law

v = velocity of flow $\propto \frac{h_L}{L}$

$v \propto i$

$v \propto \frac{h_L}{L}$; i = Hydraulic gradient

$v = ki$
↳ coefficient of permeability / hydraulic conductivity

Rate of flow $= q = vA = kAi$
↳ v = Discharge velocity = superficial velocity

$q = V_s A_v$ (V_s = actual velocity / seepage velocity)

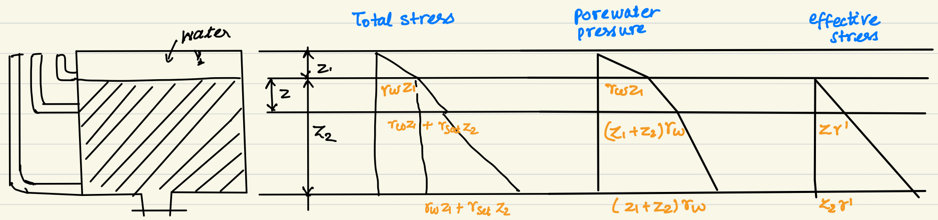
$V_s A_v = vA$

$\frac{V_s}{V} = \frac{A}{A_v} = \frac{\text{Total volume of soil}}{\text{Volume of voids}} = \frac{1}{n}$

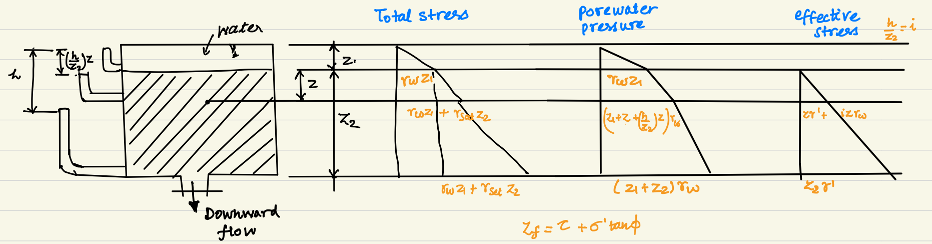
$$\boxed{V_s = \frac{v}{n}}$$ (n = porosity)

Effective Stress under Different Flow Condition

No Flow Condition

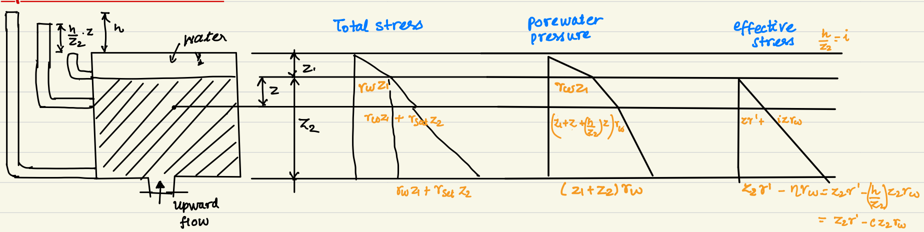


Downward Flow Condition



$\tau_f = c + \sigma' \tan\phi$

Upward Flow Condition



$z_2\gamma' - h\gamma_w = z_2\gamma' - \left(\frac{h}{z_2}\right) z_2 \gamma_w$
$= z_2\gamma' - i z_2 \gamma_w$

Quicksand Condition (Sand Boiling Condition)

Effective stress becomes equals to zero.

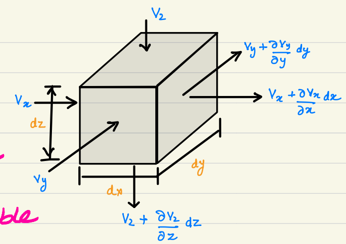
$z_2\gamma' - i z_2 \gamma_w = 0$

$\Rightarrow$ $$\boxed{i_{cr} = \frac{\gamma'}{\gamma_w}}$$
↳ Critical Hydraulic Gradient

* $i < i_{cr} \rightarrow$ Safe
* $i \geq i_{cr} \rightarrow$ Unsafe

$$\left[\text{Factor of safety} = \frac{i_c}{i_{exit}}\right]$$

3D Flow Equation



Assumptions:

(1) Soil is fully saturated & Darcy's law valid
(2) Soil is homogenous and isotropic
(3) Soil solids & the pore fluids are incompressible
(4) Flow conditions don't change with time (Steady State flow conditions exist)

Amount of water entering into element per unit time

$$q_i = V_x\,dy\,dz + V_y\,dx\,dz + V_z\,dy\,dx \longrightarrow (1)$$

Amount of water leaving the element per unit time

$$q_{out} = \left(V_x + \frac{\partial V_x}{\partial x}dx\right)dy\,dz + \left(V_y + \frac{\partial V_y}{\partial y}dy\right)dx\,dz + \left(V_z + \frac{\partial V_z}{\partial z}dz\right)dy\,dx \longrightarrow (2)$$

Assumption (1) & (3), From eqⁿ (1) & (2),

$$\Rightarrow \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)dx\,dy\,dz = 0 \longrightarrow (3)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

$$K_{x_1} = K_{x_2} = K_{x_3} = \cdots = K_{x_n}$$

$$\frac{\partial V_x}{\partial x} = K_x \frac{\partial^2 h}{\partial x^2}$$

$$\frac{\partial V_y}{\partial y} = K_y \frac{\partial^2 h}{\partial y^2}$$

$$\frac{\partial V_z}{\partial z} = K_z \frac{\partial^2 h}{\partial z^2}$$

} Homogenous Condition

(4) $\Rightarrow$ $$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$$

Isotropic $K_x = K_y = K_z$

$$\Rightarrow \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Laplace 3D Flow Equation

Velocity Potential Function (Φ)

$$\frac{\partial \phi}{\partial x} = -V_x$$

$$\frac{\partial \phi}{\partial z} = -V_z$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0 \Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

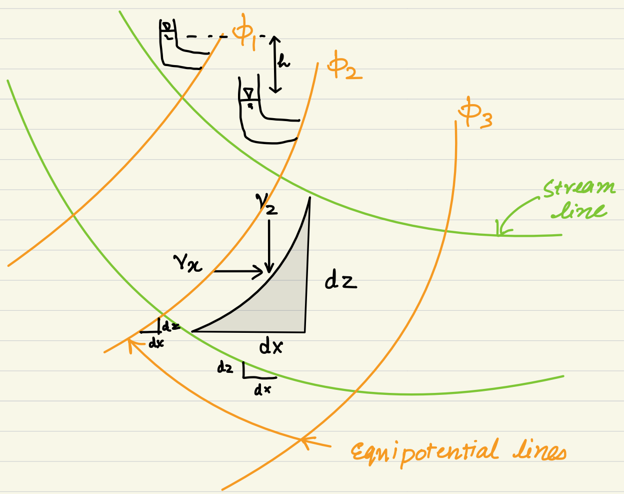
↳ Laplace Equation in terms of ϕ

Stream Function (Ψ)

$$\frac{\partial \psi}{\partial x} = -V_z \qquad \frac{\partial \psi}{\partial z} = V_x$$

or

$$\frac{\partial \psi}{\partial x} = V_z \qquad \frac{\partial \psi}{\partial z} = -V_x$$



$$dq = V_x dz - V_z dx$$
$$= \frac{\partial \psi}{\partial z} dz + \frac{\partial \psi}{\partial x} dx$$
$$= \partial \psi$$

Flow Field

$$\text{Slope of } \Phi \text{ line} = \frac{dz}{dx} = \frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial z}} = \frac{-V_x}{-V_z} = \frac{V_x}{V_z}$$

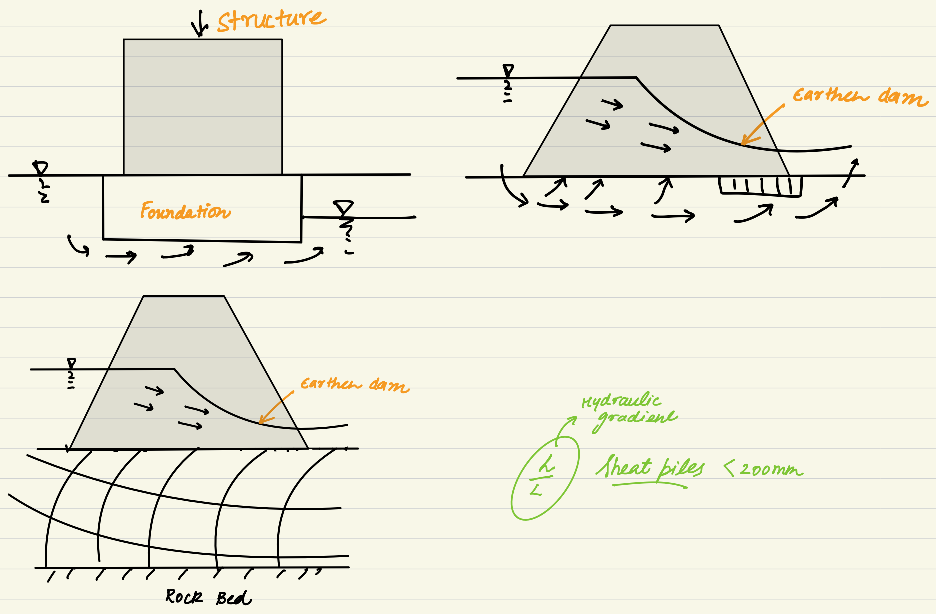
$$\text{Slope of } \Psi \text{ line} = \frac{\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial z}} = \frac{-V_z}{V_x}$$

Flow Net

A grid obtained by drawing a series of equipotential lines & flow lines orthogonal to each other.

Uses:

(1) Determination of seepage loss from Reservoir
(2) Determination of seepage pressure or uplift pressure
(3) Determination of exit gradient to check the possibility of piping failure below dam or structure.



Hydraulic gradient $\frac{h}{L}$

Sheat piles < 200mm

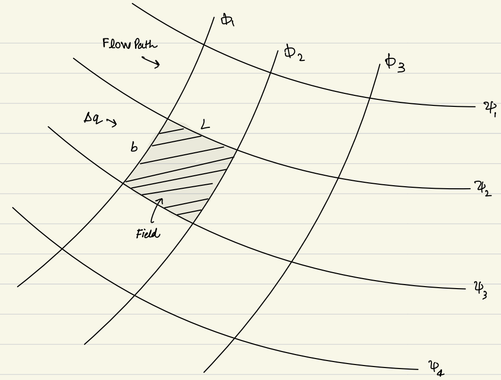
N_f = No. of flow paths
N_d = No. of potential drops
Δh = head loss/ potential drop
H = Total loss of head b/w entry and exit point
$= (\Delta h)\, N_d$

Δq = the discharge through one flow path

$= k i A$

$= k \frac{\Delta h}{Z} (b \times 1)$

$= k \frac{H}{N_d} \left(\frac{b}{L}\right)$



$(N_d = \phi_n - 1)$

q = Total discharge through the flow space
$= K N_f \frac{H}{N_d} \left(\frac{b}{\ell}\right) = \frac{K H N_f}{N_d}$ $(b = \ell)$

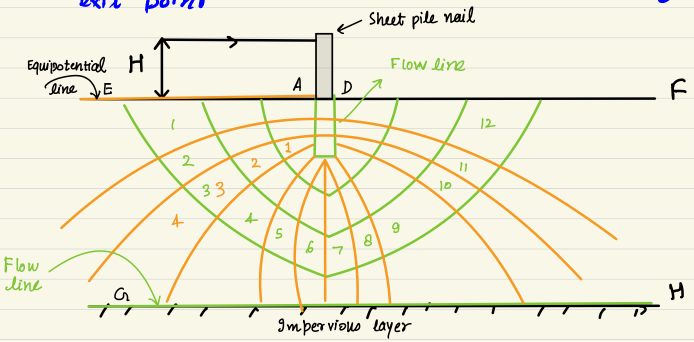
$\gamma' z$
$\gamma' z + i z \gamma_w$
$\gamma' z - i z \gamma_w$

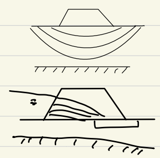
Seepage frce / vol. $= \frac{i z \gamma_w A}{z A} = i \gamma_w$

Construction of Flownets

Properties :

(1) Equipotential lines & Flow lines are orthogonal to each other.
(2) The flow field should be approximatly square.
(3) The curves should be smooth (Parabolic / Elliptical)
(4) The size of squares should change gradually from entry to exit point



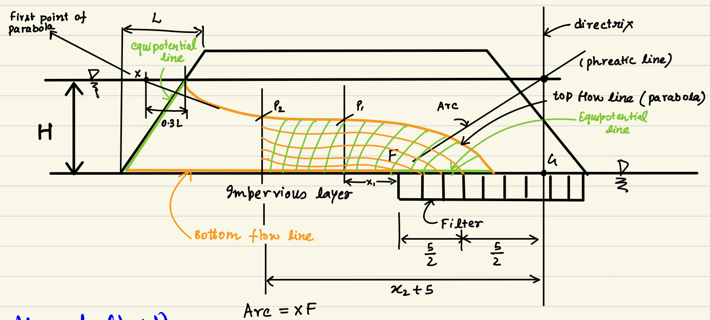


$q = K H \frac{N_c}{N_d}$ $N_c = 4$, $N_d = 12$

All these fields should be approx. square shaped.

Flownet for Discharge Calculation through the body of Earthen Dam

A. Casagrande



$P, F = HG$ (unit length of flow)

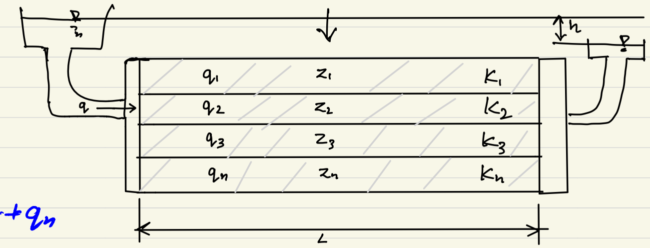
$q = K \frac{dz}{dx} z = K \frac{s}{(s^2 + 2s x)^{1/2}} (s^2 + 2xs)^{1/2} = 2ks$

$FP = \sqrt{x^2 + z^2} = PG = x + 5$

$\Rightarrow z = (x^2 + s^2 + 2xs - x^2)^{1/2} = (s^2 + 2xs)^{1/2}$

$\frac{dz}{dx} = \frac{s}{(s^2 + 2xs)^{1/2}}$

Coefficient of Permeability of Stratified Soil

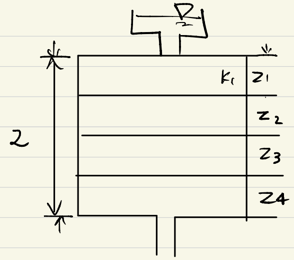


$$q = q_1 + q_2 + q_3 + \cdots + q_n$$

$$kiA = k_1 i_1 A_1 + k_2 i_2 A_2 + \cdots + k_n i_n A_n$$

$$kz = k_1 z_1 + k_2 z_2 + \cdots + k_n z_n$$

$$\boxed{k = \frac{k_1 z_1 + k_2 z_2 + \cdots + k_n z_n}{z}}$$



A = c/s area of flow

q = " "

$$q = q_1 = q_2 = q_3 = q_n$$

h = Total loss of head $= h_1 + h_2 + h_3 + \cdots + h_n$

$$V = \frac{Kh}{z} = \frac{K_1 h_1}{z_1} = \frac{K_2 h_2}{z_2} = \cdots = \frac{K_n h_n}{z_n}$$

$$\Rightarrow h_1 = \frac{Vz_1}{K_1},\ h_2 = \frac{Vz_2}{K_2},\ \ldots,\ h_n = \frac{Vz_n}{K_n}$$

$$h = h_1 + h_2 + h_3 + \cdots + h_n$$

$$\frac{Vz}{K} = \frac{V_1 z_1}{K_1} + \frac{V_2 z_2}{K_2} + \cdots + \frac{V_n z_n}{K_n} \Rightarrow \boxed{K = \frac{z}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \cdots + \frac{z_n}{K_n}}}$$

Flow through Anizotropic Soil

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0, \quad K_x \neq K_z$$

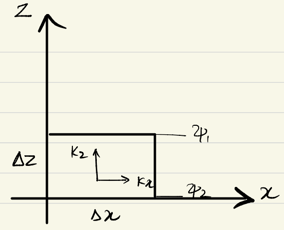
equation is not satisfying laplace's condition $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$

$$\Rightarrow \frac{\partial^2 h}{\frac{K_z}{K_x}\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{—— (2)}$$

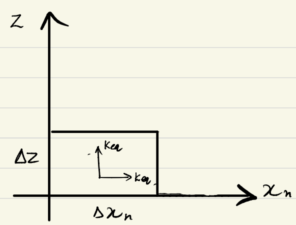
$$\boxed{x_n = \left(\sqrt{\frac{K_z}{K_x}}\right) x}$$

$$\Rightarrow \frac{\partial^2 h}{\partial x_n^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{—— (3)}$$

Eqn (3) represents flow condition in x_n - z coordinate



Actual section (Anisotropic)



Transformed section (Equivalent isotropic soil)

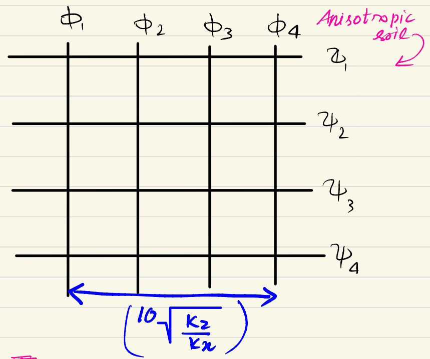
$$\Delta x_n = \left(\sqrt{\frac{K_z}{K_x}}\right) \Delta x$$

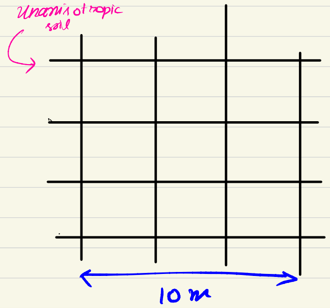
$$\Delta q = K_x \frac{\Delta H}{\Delta x} \Delta z \quad \text{(Anisotropic) —— (4)}$$

$$\Delta q = K_{eq} \frac{\Delta H}{\Delta x_n} \Delta z \quad \text{(isotropic) —— (5)}$$

Equating (4) and (5)

$$K_x \frac{\Delta H}{\Delta x} \Delta z = K_{eq} \frac{\Delta H}{\Delta x_n} \Delta z \Rightarrow \boxed{K_{eq} = \sqrt{K_x K_z}}$$





There is no change in vertical dir^n length

There is change only in horizontal dir^n length

* In few cases, $K_z > K_x$ (loess)

$q = KS$

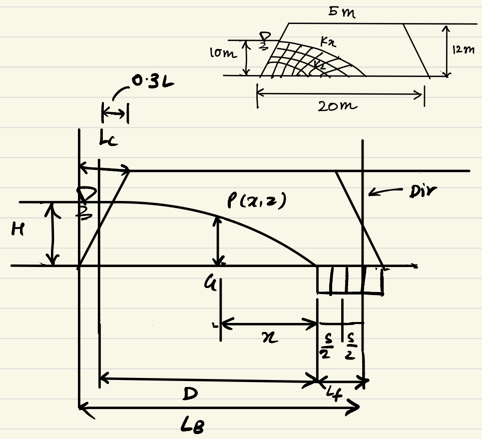
$D = L_B - L_f - 0.7L$

$\sqrt{x^2 + z^2} = x + s$

$\Rightarrow s = \sqrt{x^2 + z^2} - x$

when $x = D$, $z = H$

$$S = \left(\sqrt{D^2 + H^2}\right) - D$$



Flow through non-homogenous soil

θ_1 = Angle of intersection of streamlines at the interface

θ_2 = Angle of deflection of stream lines at the interface

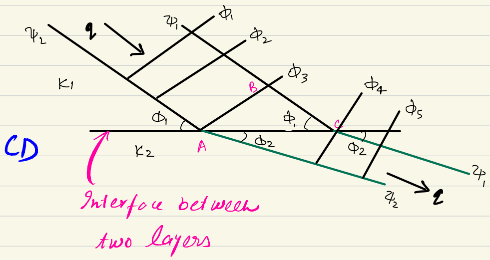
$$q = K_1 \left(\frac{\Delta h}{\frac{BC + D}{2}}\right) AB = K_2 \frac{\Delta h}{\left(\frac{AB}{2}\right)} CD$$

$$\Rightarrow K_1 \frac{AB}{BC} = K_2 \frac{CD}{AD}$$

$$\Rightarrow K_1 \tan\theta_1 = K_2 \tan\theta_2$$

$$\Rightarrow K_1 = K_2 \frac{\tan\theta_2}{\tan\theta_1}$$

How net can be drawn



Design of Graded Filter

→ used to permit the flow of seepage water without allowing the movement of soil particles.
→ prevents the erosion / piping failure of soil
→ mainly of coarse grained soil (sand & gravel)

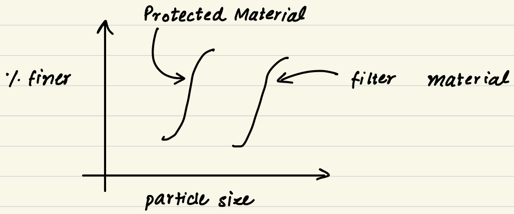
Criteria — Free draining soil

(1) $\frac{D_{15}\ (\text{Filter Material})}{D_{85}\ (\text{Protected Material})} < 5$

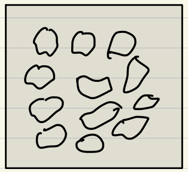
(2) $4 < \frac{D_{15}\ (\text{filter})}{D_{15}\ (\text{protected})} < 20$

(3) $\frac{D_{50}\ (\text{Filter})}{D_{50}\ (\text{protected})} < 25$

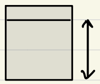
(4) The grain size distribution curve of the filter material should be approximately parallel to GSO curve of protected material.



Compaction



P.R Proctor (1933)
Standard Proctor Test

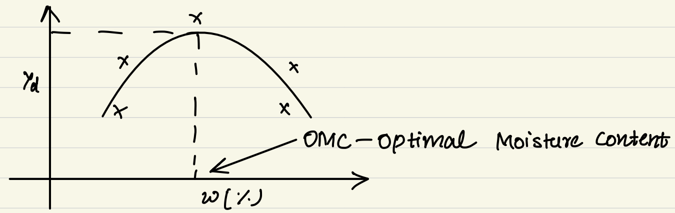


Dry condition

$\gamma = \gamma_d = \frac{W_d}{V}$

$\gamma_d = \frac{W_{dry\ soil}}{V}$

$\gamma = \frac{W}{V}$



Samples w_1 w_2 w_3

$\gamma = \frac{W}{V}$

$\gamma_d = \frac{\gamma}{1+w}$

$e = \frac{G\gamma_w}{\gamma_d} - 1$

Theoritical Mass unit weight

$(S = 100\%)$ $\gamma_{d\,max} = \frac{G\gamma_w}{1 + wG/S_r}$

$n_f = \frac{V_0}{V}$

$\gamma_{d_{max}} = \frac{(1-n_0)G\gamma_w}{1+wG}$

5% are valid line, 95% saturation line

$\frac{0.95\gamma_w G}{0.95 + wG}$

Light Compaction :- Compaction of backfill material behind retaining walls, foundation soil below lightly loaded structures.

Heavy Compaction :- Foundational soil below heavily loaded structures – Airfield, Dams.

Standard Proctor Test to simulate the light compaction condition of the field

For heavy compaction, Modified Proctor Test used.

MPT – Size of mould same as SPT (Standard Proctor Test)

Wt. of hammer = 4.89 kg
Height of drop = 450 mm
No. of layers = 5
No. of blows = 25
(discard)

Sieve
20 mm
4.75

↑ Retained
(>20%) if yes

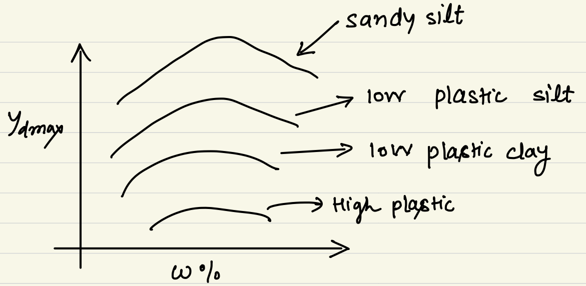
(Mould = 150 mm)
[No. of blows = 56]

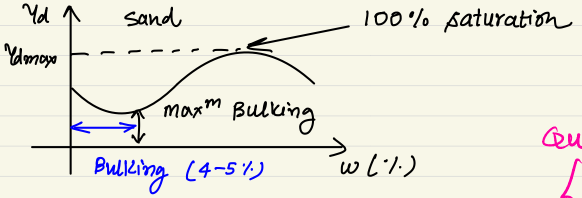
Compactive Effect

It is defined in terms of compaction energy per unit vol.

$$E = \frac{(\text{No. of blows per layer}) \times (\text{No. of layers}) \times (\text{Wt. of hammer})}{\text{vol. of mould}}$$

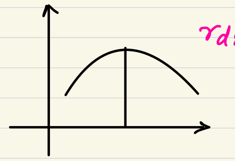
Parameters affecting the compaction of soil
(1) Water Content
(2) Compactive effect
(3) Types of soil
(4) Method of Compaction

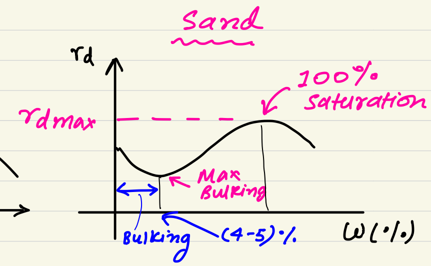




Questions for Midsem
- Theoritical Questions
- Numericals
- Derivations

Bulking of Sand





The increase in volume of sand with increase in M.C. at a given compactive effort.

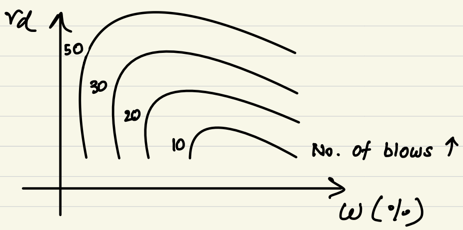
⇒ Development of capillary tension in the pore water.
⇒ This capillary tension offers resistance against the compactive effort. Thus, instead of decrease in volume, the volume of sand increases.

* Generally we get max. density at this 100% saturation point.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$\frac{w_L - w_A}{w_L - w_P}$$

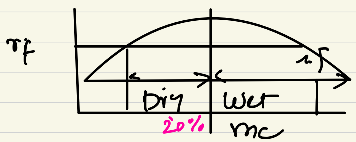
Used to define the denseness of sand
loose < 35%
Medium dense (35-70)%
Dense > 70%



* Different kinds of Roller & their compaction sands

Method of Compaction

(1) Rolling Action
(2) Tomping Action
(3) Kneading Action
(4) Vibrating Action



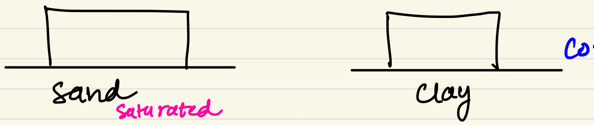
Placement Water content or Moisture content

* Dry of optimum (w < omc)
* wet of optimum (w > omc)

Relative compaction $= \frac{\gamma_d}{\gamma_d}$ → attained at the field / → obtained from lab <9

$\gamma_d = \frac{\gamma}{1+w}$ 90 to 95 %

* Proctor Tensile → * Calibration charge !



Consolidation of soil

Dry Soil ⇒ Settlement occurs due to compression of air or removal of air.

Saturated ⇒ There is no air → water will flow in lateral dir^n bcoz of its tendency.

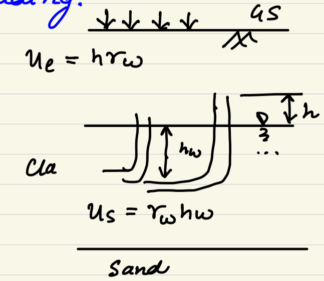
* linted
* DPC ⇒ Damp pressure course !!

Consolidation

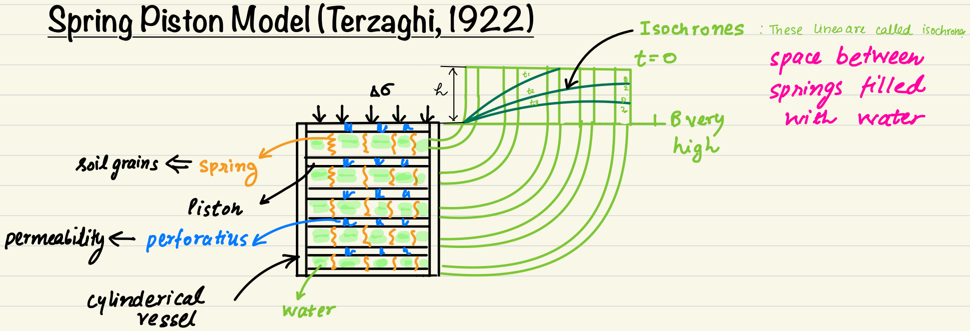
The process of change of volume of soil due to expulsion of water under transient flow condition from voids which occurs on account of dissipation of excess pore water pressure under sustained / constant static loading.

Seepage Flow :

Transient Flow :



Spring Piston Model (Terzaghi, 1922)



space between springs filled with water

(1) Immediate after application of loading, there is no compression in the spring. Thus, the total load B carried by the water (t=0)

$u_e = \Delta\sigma = h\gamma_w$

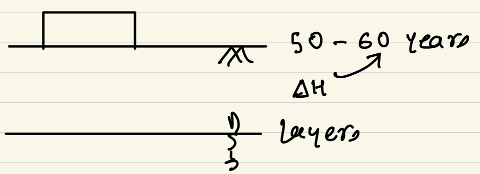
(2) With the passage of time, the water is expelled from the cylinder and the load is transferred to the springs.

(3) At t is very high, $u_e = 0$, all the load is carried by the spring.

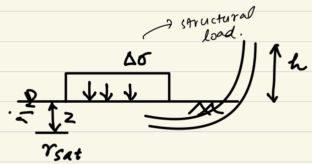
Degree of consolidation $= \frac{\Delta h}{\Delta H} \times 100$

Δh = the deformation / compression of the spring at any time 't'

ΔH = Total " / " at end of consolidation.



Just before loading



$\sigma = z\gamma_{sat}$ $u_s = z\gamma_w$

$\sigma' = \sigma - u_s = z\gamma_{sat} - z\gamma_w$

Immediate after application of loading

$\sigma = z\gamma_{sat} + \Delta\sigma$ $u_s = z\gamma_w$ $u_e = h\gamma_w = \Delta\sigma$

$\sigma' = z\gamma_{sat} + \cancel{\Delta\sigma} - (u_s + \cancel{u_e})$

$= z\gamma_{sat} - z\gamma_w$

At the end of dissipation of excess pore water.

$\sigma = \gamma_{sat} z + \Delta\sigma$

$u_s = \gamma_w z$

$u_e = 0$

$\sigma' = \gamma_{sat} z + \Delta\sigma - \gamma_w z$

The change in volume of soil /consolidation depends in the effective stress on soil due to dissipation of excess pore water pressure.

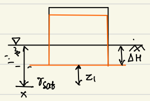
(1) Immediate consolidation (coarse grained soil)
(2) Primary consolidation
(3) Secondary consolidation

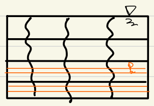
Consolidation

$\sigma = \gamma_{sat} z + \Delta\sigma$

$\sigma' = \gamma_{sat} z + \cancel{\Delta\sigma} - (u_s + \cancel{u_e})$

$= \gamma_{sat} - (u_s = \gamma_w z)$



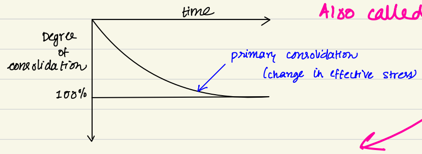


$\sigma' = \gamma_{sat} z_1 + \Delta\sigma - (u_s + \cancel{u_e}^{\,0})$

$\sigma = \gamma_{sat_1} z_1 + \Delta\sigma$

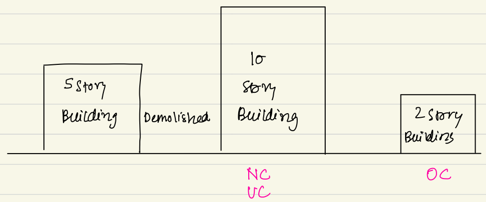
* Total stress remains constt.

(1) Immediate consolidation]→ Sand (coarse grained soil)
↳ Immediately all the water will drain out
(2) Primary consolidation]
(3) Secondary consolidation]→ Clayey (fine grained soil)
Also called creep consolidation



* Secondary consolidation occurs due to plastic readjustment of the solid grains once the primary consolidation is over.
* Effective stress remains constant.
* Also called creep.

* primary consolidation occurs due to dissipation of excess pore water pressure

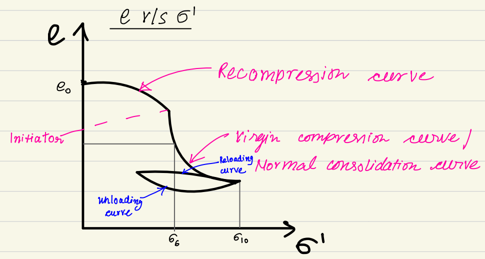


Depending on the stress history, the soil is divided into

(1) Normally consolidated : The soil in which the present normal effective stress is the maximum stress it has ever been experienced in its stress history. (The consolidation is already over at the present state of stress)

(2) Over consolidated : present state of stress is less than the maximum stress it has ....

(3) Under consolidated : The soil is yet to consolidate under the present effective stress.

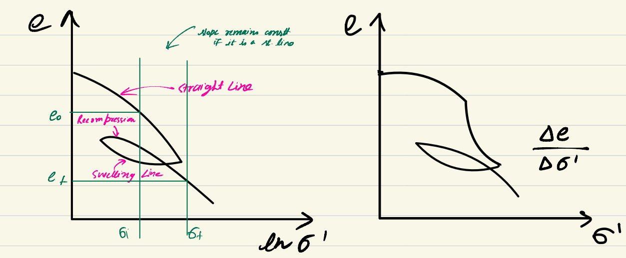


* In over consolidated soil OCR > 1
* In Normal Consolidated Soil OCR = 1

$OCR = \text{Over Consolidation ratio} = \dfrac{\sigma_c'}{\sigma'}$

σ' = present effective normal stress

σ_c' = pre consolidation effective stress



Change in volume of soil = change in volume of voids.

coefficient of compressibility $(a_v) = \frac{-\Delta e}{\Delta \sigma'}$

coefficient of volume compressibility $(m_v) = \frac{-\Delta V}{V_0} \frac{1}{\Delta \sigma'}$

$$\frac{\Delta V}{V_0} = \frac{V_{V_f} - V_{V_0}}{V_s + V_{V_0}} = \frac{\left(\frac{V_{V_f} - V_{V_0}}{V_s}\right)}{\left(\frac{V_s + V_{V_0}}{V_s}\right)} = \frac{e_f - e_0}{1+e} = \frac{\Delta e}{1+e}$$

(ΔV: Total initial volume; V_{V_f}: Final volume of voids; V_{V_0}: Initial volume of voids; V_s: Volume of solid; e_f: Final void ratio; e_0: Initial void ratio)

$$m_v = \frac{\Delta e}{1+e_0} \frac{1}{\Delta \sigma'} = \frac{a_v}{1+e_0}$$

$$\frac{\Delta V}{V_0} = \frac{\Delta e}{1+e} \Rightarrow \frac{\Delta H\, A}{H A} \Rightarrow \frac{\Delta V}{V_0} = \frac{\Delta H}{H_0}$$

(ΔH: change in ht; H: change in height before loading)

$$m_v = \frac{\Delta e}{1+e_0} \frac{1}{\Delta \sigma'}$$

$$\Rightarrow m_v = \frac{\Delta H}{H} \frac{1}{\Delta \sigma'}$$

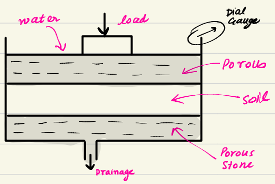
$$\Rightarrow \boxed{\Delta H = m_v H \Delta \sigma'}$$

Slope of NCC (Compression Index) $= C_c = \frac{\Delta e}{\ln\left(\frac{\sigma_f'}{\sigma_0'}\right)} = \frac{\Delta e}{\ln\left(\frac{\sigma_0' + \Delta \sigma'}{\sigma_0'}\right)}$

$$\frac{\Delta H}{H} = \frac{\Delta e}{1+e_0} \Rightarrow \Delta H = \frac{H C_c}{1+e_0} \ln\left(\frac{\sigma_0' + \Delta \sigma'}{\sigma_0'}\right)$$

Applied Stress	Final Dial Reading	Change in Dial Reading ΔH	Height of sample $H_0 + \Delta H = H$
0	1.25		25 mm
6.25	1.15	x_1	24.5 mm
25		x_2	23
50	decrease		
100			
200			20 mm
400			
200			25 mm
100			
50			22.5 mm → e_f =

Here in this test we are assuming 100% saturation.



$$e = \frac{V_v}{V_s} = \frac{H_0 - H_s}{H_s}$$

(H_s: Height of soil solids)

$W_s = \gamma_s V_s$ (unit wt of soil solids; vol. of soil solids)

$W_s = G \gamma_w H_s A$

$$H_s = \frac{W_s}{G \gamma_w A} = \frac{W_d}{G \gamma_w A}$$

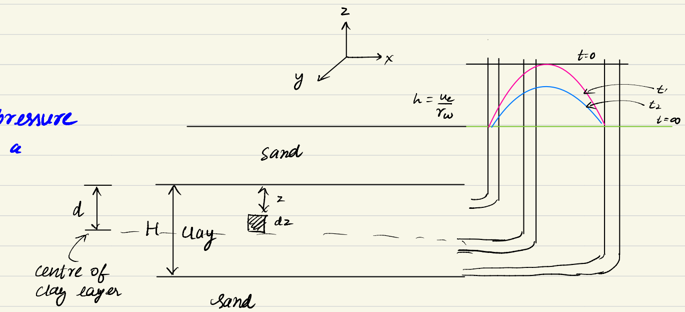
AFTER MIDSEM →

Solution of one dimensional Consolidation Equation

$$\frac{\partial u_e}{\partial t} = C_{v_z} \frac{\partial^2 u_e}{\partial z^2}$$

u_e = excess pore water pressure at any time 't' at a depth 'z'.

$$C_{v_z} = \left(\frac{1+e_0}{a_v \gamma_w}\right) K_z$$



(1) $t = 0$, $u_i = u_e$ (same at all the locations)

(2) $t = \infty$, $u_e = 0$

(3) $t > 0$
where $z = 0$ $u_e = 0$
$z = H$ $u_e = 0$

Using Fourier series expansion & putting the boundary conditions.

$$u_e = \sum_{n=1}^{n=\infty}\left(\frac{1}{d}\int_0^{2d} u_i \sin\left(\frac{n\pi z}{d}\right)dy\ \sin\left(\frac{n\pi z}{2d}\right)\exp\left(\frac{n^2\pi^2 c_v t}{4d^2}\right)\right)$$

where n is an integer.

(1) n = even $1-\cos n\pi = 0 \Rightarrow u_e = 0$

n = odd $1-\cos n\pi = 2$

Putting the above conditions

$$u_e = \sum_{n=1}^{n=\infty}\frac{2u_i}{n\pi}(1-\cos n\pi)\sin\left(\frac{n\pi z}{2d}\right)\exp\left(-\frac{n^2\pi^2 c_v t}{4d^2}\right)$$

Putting $m = 2n+1$, where n is an integer. $T_v = \frac{c_v t}{d^2}$

$$u_e = \sum_{m=0}^{m=\infty}\frac{2u_i}{(2n+1)\pi}(1-\cos(2n+1)\pi)\sin\frac{(2n+1)\pi z}{2d}\exp\left(-\frac{(2n+1)^2\pi^2 T_v}{4}\right)$$

$$= \sum_{m=0}^{m=\infty}\frac{2u_i}{M}\sin\left(\frac{Mz}{d}\right)\exp(-M^2T_v)$$

$M = \frac{(2n+1)\pi}{2}$ → constant, $T_v = \frac{c_v t}{d^2}$ = Time Factor

$d = H$ (Single Drainage)

$d = H/2$ (Double Drainage)

d = distance of drainage path = the maximum distance travelled by water particle to reach the free drainage layer

Degree of consolidation at any depth 'z' at any 't'

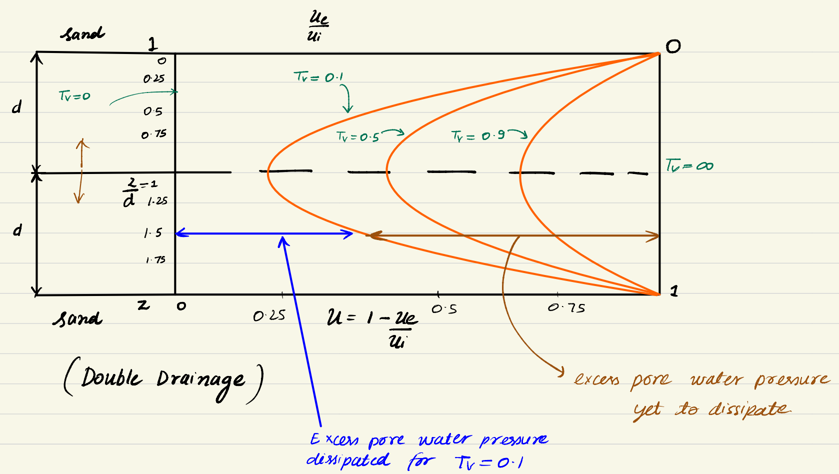
$$U = \frac{\text{excess pore water pressure dissipated}}{\text{initial excess pore water pressure}} = \frac{\Delta u_e}{u_i} = \frac{u_i - u_e}{u_i} = \frac{\Delta\sigma'}{u_i}$$

$$= 1 - \frac{u_e}{u_i}$$

$\Delta\sigma'$ = effective stress transferred to the soil due to dissipation of excess pore water pressure = Δu_e

$$U(z,t) = 1 - \sum_{m=0}^{m=\infty}\frac{2}{M}\sin\left(\frac{Mz}{d}\right)\exp(-M^2T_v)$$

$u(z,t) = 1 - f(z)f(t)$

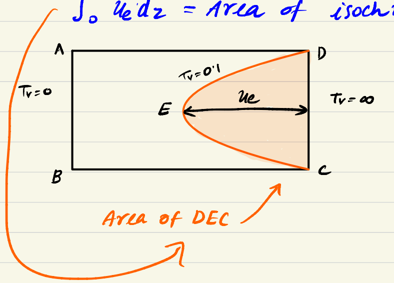


Average degree of consolidation of a layer

$$U_{av} = \frac{\frac{1}{2d}\int_0^{2d} u_i\,dz - \frac{1}{2d}\int_0^{2d} u_e\,dz}{\frac{1}{2d}\int_0^{2d} u_i\,dz}$$

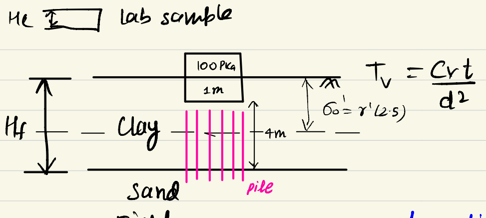
$\int_0^{2d} u_i\,dz$ = Area of isochrone under initial excess porewater pressure

$\int_0^{2d} u_e\,dz$ = Area of isochrone under initial excess porewater pressure at any time 't'



$$U_{av} = 1 - \sum_{m=0}^{m=\infty}\frac{2}{M^2}\exp(-M^2T_v)$$

$U_{av} = f(T_v)$



$T_v = \frac{C_v t}{d^2}$

t_f = time required to attain a certain degree of consolidation = U %

t_ℓ = " " " " " " at fild = U%

$T_{v\,lab} = T_{v\,field}$

$C_{v\,lab} = C_{v\,field}$

d_ℓ = drainage path at lab ($d_\ell = H_L$ for single drainage, $d_\ell = H_L/2$ for double drainage)

d_f = " " at field ($d_f = H_f$ for " ", $d_f = H_f/2$ " ")

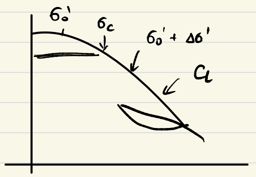
$\sigma_0' > \sigma_c'$ (NC)

$\sigma_0' < \sigma_c'$ (OC)

$\sigma_0' + \Delta\sigma' < \sigma_c'$

$\sigma_0' < \sigma_c' < \sigma_0 + \Delta\sigma'$

$\frac{t_L}{d_L^2} = \frac{T_f}{d_f^2} \Rightarrow T_f = ?$

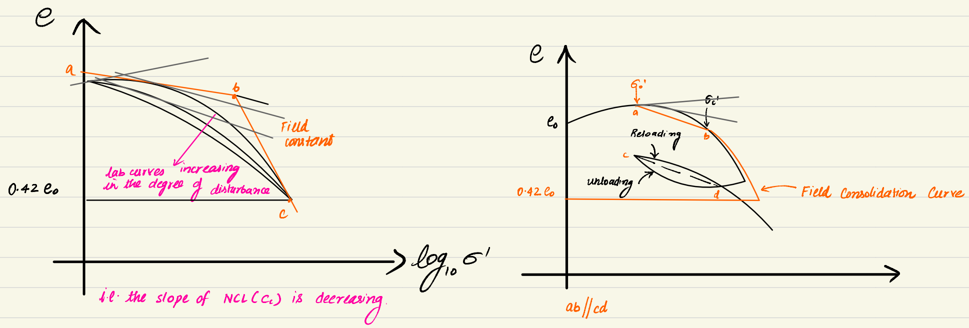


$\Delta H = \frac{C_r H}{1+e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma}{\sigma_0'}\right)$

Doubtful eqn check once!

$\Delta H = \frac{C_r H}{1+e_0} \log_{10}\left(\frac{\sigma_c'}{\sigma_0'}\right) + \frac{C_c H}{1+e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_c'}\right)$

Extrapolation of field consolidation curve from lab consolidation curve (Schmertman, 1955)



The deformation / volume change due to consolidation has two aspects:-

(1) The amount of volume change will occur.

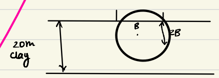
(2) How long will it take for the volume change to occur.

→ Depends

(a) How much stress is applied i.e. loading condition.

(b) How much soil is affected i.e. the zone of influence

(c) How compressible is the soil i.e. the property of soil (C_c)



Depends on

(a) Amount of volume change to occur

(b) Number and location of free Draining layers.

(c) Permeability of soil i.e. property of soil.

K, E } Engineering property of soil

liquid limit of soil.

$C_c = 0.007 (w_L - 10)$ [Removable soil sample]

$C_c = 0.009 (w_L - 10)$ [undisturbed " "]

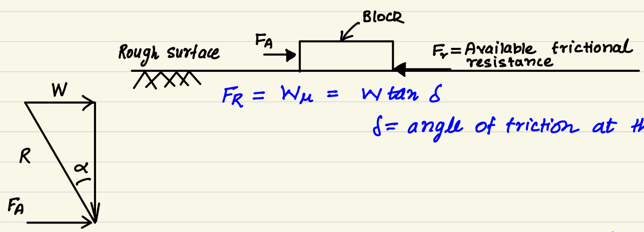
w_L – Liquid limit of soil

Shear Strength of Soil

Soil derives its shear strength from the following:-

1) Resistance due to interlocking between the particles.
2) Frictional Resistance between the particles (sliding friction, Rolling friction, both)
3) Cohesion (Bonding) between the particles

Frictional Resistance Concept



$F_R = W\mu = W \tan\delta$

δ = angle of friction at the interface

α = Angle of Obliquity

when $F_A = F_R$ (α is maximum)

$W \tan\alpha = W \tan\delta$

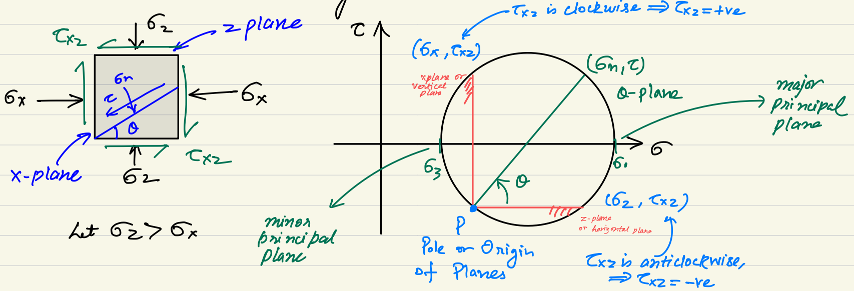
$\alpha = \delta$

$\alpha_{max} = \phi$

φ = Angle of friction in soil mass.

Applicable when the soil is purely frictional (c = 0)

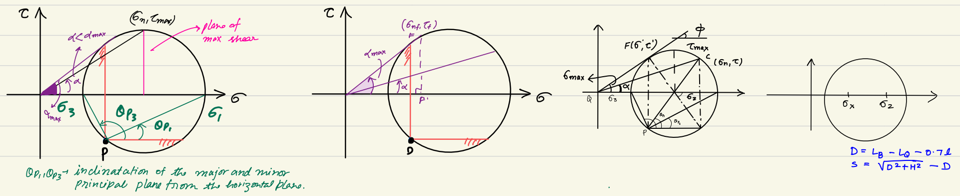
Consider the following state of stress



Let $\sigma_z > \sigma_x$

- Using (σ_x, τ_{xz}) and (σ_z, τ_{xz}) the pole P is located in the Mohr Stress Circle.
- From P, the line can be drawn to find out stresses at any plane

{Clockwise → +ve
Anticlockwise → −ve}



Frictional Material (Sand, Gravel)

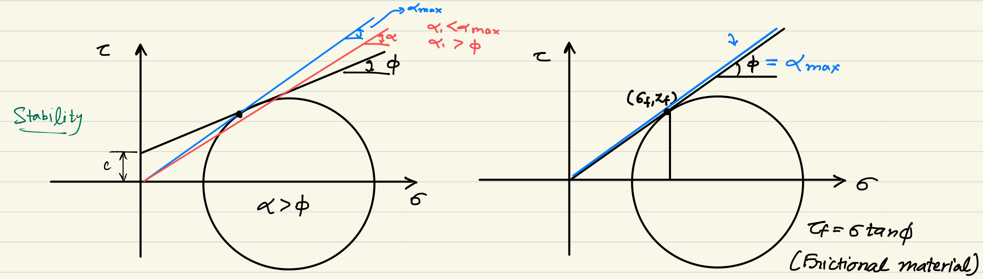
$\tau_f = \sigma \tan\phi$ (Mohr's Theory, 1886)

(σ → Normal stress; ϕ → Angle of friction)

Shear Stress @ failure = Shear Strength of the material

$\tau_f = c + \sigma \tan\phi$ (Coulomb's Theory, 1776)

(c → cohesion of material)

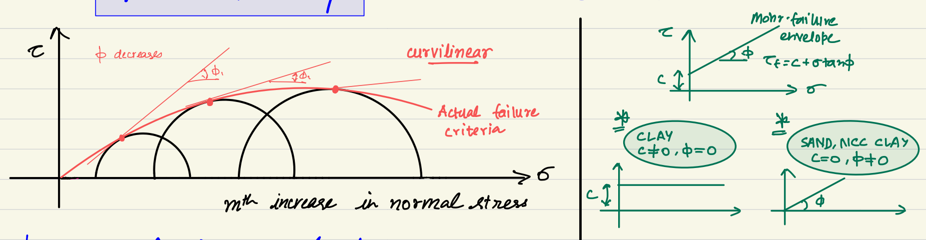


c and ϕ are the shear strength parameters of the soil.

Mohr Coulomb Failure Criteria

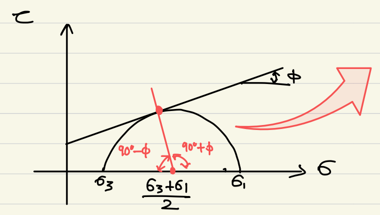
$$\tau_f = c + \sigma \tan\phi$$

- Actual failure criteria is curvilinear
- Linearized.



- ϕ ↓ with ↑ in normal stress
- Mohr's Circle shifts and hence the yield line also converges ⇒ ϕ ↓
- ϕ is decided based on the σ range in the actual field condition.
  for lower (σ_3, σ_1), ϕ is higher ⇒ τ_f is higher → overestimates the strength
- c & ϕ are not inherent property of the material.
  - c & ϕ → shear strength parameters of the soil.
  - depends on * loading * drainage * degree of saturation / moisture content

→ EXTRA NOTES



In real life, angle is always half

MAJ → $45° + \phi/2$
MIN → $45° - \phi/2$

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\sigma_3 = \sigma_1\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

} Relationship b/w σ_1 and σ_3

* If σ_1 and σ_3 are known, and any plane AB inclined to angle θ to the dirn of major principal plane, the normal stress σ and shear stress 'τ'.

$$\sigma = \left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta$$

$$\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\theta$$

* The sum of principal stresses on mutually perp. plane is constt.

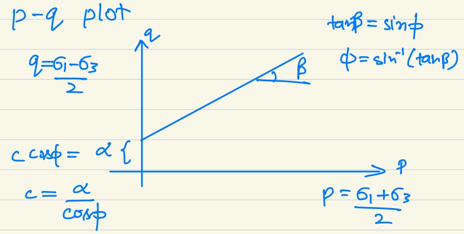
$$\sigma_1 + \sigma_3 = \sigma_{n_1} + \sigma_{n_2} = \text{constt.}$$

→ EXTRA NOTES

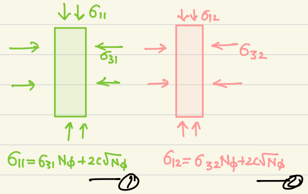
→ Determination of c & φ from p-q plot

$$\left[\frac{\sigma_1-\sigma_3}{2}\right] = \left[\frac{\sigma_1+\sigma_3}{2}\right]\sin\phi + c\cos\phi$$

(first bracket → q; second bracket → p)



→ Let $N_\phi = \frac{1+\sin\phi}{1-\sin\phi}$ then



Two eqn → Two unknown N_ϕ, c

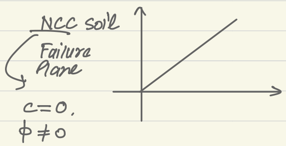
⇒ N_ϕ and c can be found out.

* Drained ⇒ $\Delta U = 0$

Undrained ⇒ $\Delta U \neq 0$

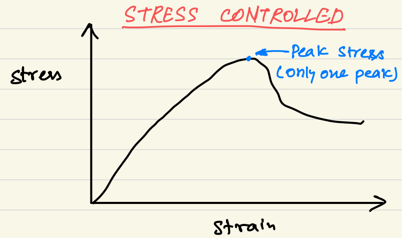
* $\phi = \phi'$ in case of drain test

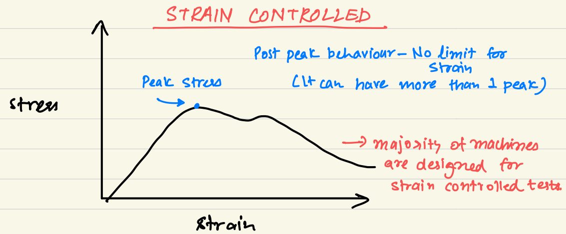
$\phi = 0$ in undrained test



Tests for determination of c & Φ

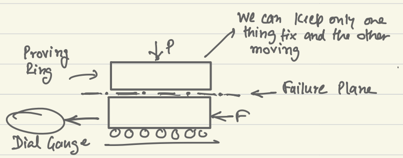
1. Direct Shear test
2. Triaxial Compression test
3. Unconfined Compression test
4. Vane Shear test

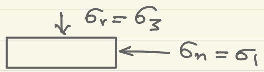




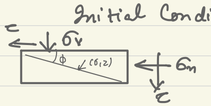
Stress controlled Test (Stress is applied at a constant rate)

Strain controlled Test (Strain is applied at a constant rate)

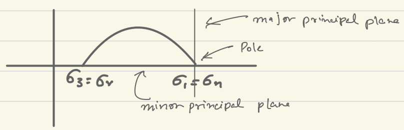


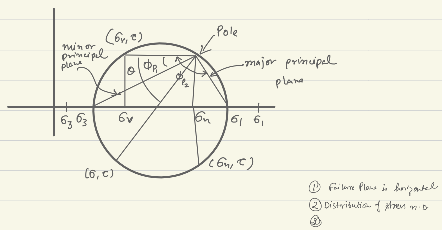


Initial Condition



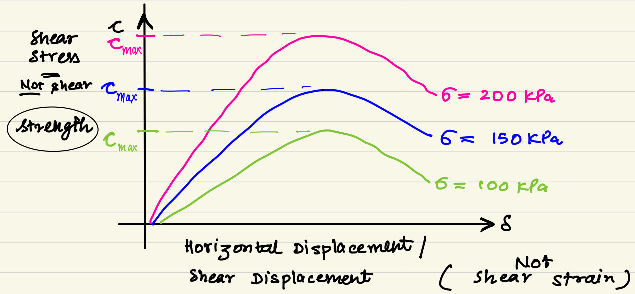
At any intermediate

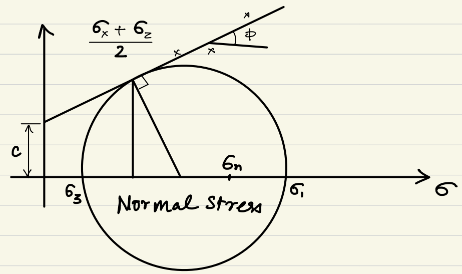




① Failure Plane is horizontal
② Distribution of stress is ...
③

Direct Shear Test





Triaxial Compression Test

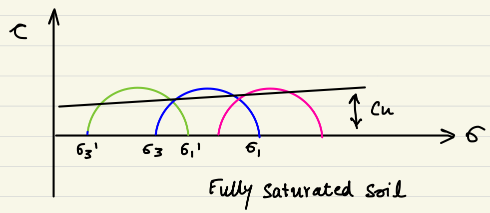
$\sigma_1 = \sigma_3 + \sigma_d$

Application of Load (Two stages)

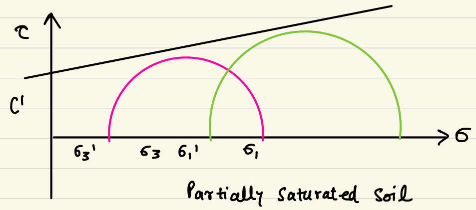
Test	Stage-1 (σ_3) — Application of confining pressure / cell pressure / All around " (No shearing)	Stage-2 (σ_d) — Application of Differential stress (shearing of soil)
UU	Unconsolidated (Drainage valve remains closed)	Undrained (Drainage valve remains closed)
CU	Consolidated (Drainage is allowed)	Undrained (Drainage valve remains closed)
CD	Consolidated (Drainage is allowed)	Drained (Drainage is allowed)

(UD) X

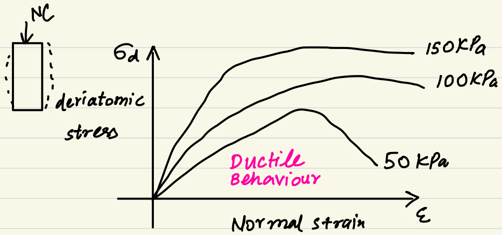
In clayey soil → Cohesion
sandy soil → No cohesion (generally)



Fully Saturated Soil



Partially Saturated Soil



c, ϕ

Back Pressure is applied to saturate the soil through the drainage pipe / a seperate pipe is connected to the soil sample.

Back pressure < Confining Pressure
6 kPa 10 kPa = 25

$\frac{\Delta u}{\Delta \sigma} = 0.5$, $\frac{\Delta u}{\Delta \sigma} = 0.9$

$\Delta u = 2$ kPa
$= 1$ kPa

Mohr Coulomb valid for any (Total course)

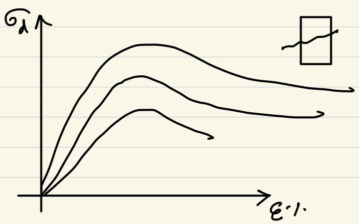
NC
Lightly OC   OC ≅ 3

$$DCR = \frac{\sigma_{experienced}}{\sigma_{present}}$$ = Heavily OC / Now

↑ applied confined pressure

$\sigma_{1f} = \sigma_{3f} + \sigma_{df}$
↓
σ_3

$\sigma'_{3f} = \sigma_{3f} - u_f$
$\sigma'_{1f} = \sigma_{3f} - u_f$



EXTRA NOTES

1st stage	2nd Stage
Drainage allowed	Volume change allowed
Drainage not allowed	Volume change not allowed

$$\gamma_t = \left(\frac{G + Se}{1+e}\right)\gamma_w$$ → $Se = wG$

$\gamma_w = 9.81$ kN/m³

$$\gamma_d = \frac{\gamma_t}{1+w}$$ → Dry unit weight.