CE361A

ENGINEERING HYDROLOGY

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Aman

Introduction

1 Aug

Hydrology:- Geoscience that describes and predicts the occurence & circulation of the earth's fresh water.

why should we study hydrology?

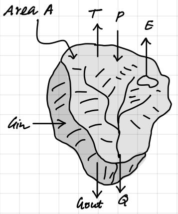
1. water resource management : meeting increasing water needs under a changing climate.
2. extreme events : floods and droughts.

Water Budget Equation

Watershed is the area that topographically contributes to flow at a common outlet.

change in storage → $\boxed{\begin{array}{l}\Delta S = \text{Inflow} - \text{Outflow} \\ \Delta S = (P + G_{in}) - (E + T + G_{out} + Q)\end{array}}$

- Units: Volume / time



Inflow { P : Precipitation
G_{in} : Groundwater inflow in the region

Outflow { E : Evaboration
T : Transpiration
G_{out} : Groundwater outflow from the region
Q : Streamflow

Application of Water Budget Equation

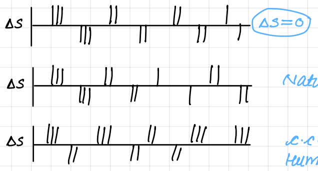
$\Delta S =$ inflow ↑ − outflow ↓

↳ water available for use

canals − inflow ↑

recharge ponds − outflow ↓

problem : we don't have many terms known.



Natural natural condition

c.c. Human long term average condition.

Topics

Introduction : Hydrologic cycle, water budget, world water quantities	1
Precipitation	3
Infiltration	2
Evaporation and Transpiration	2
Runoff and hydrographs	2
Floods, Return Period analysis and Routing	7
Groundwater Hydrology	7

Marking :- Quizzes X3 − 25 %.
Midsem − 30 %.
Endsem − 45 %.

Precipitation

3 Aug

The term precipitation includes all forms of water reaching the earth from the atmosphere (like rainfall, snowfall, hail, frost & dew).
↳ significant amount

Forms of Precibitation

Rain {
- liquid , 0.5 − 6 mm
- Rate: light (<2.5 mm/hr), moderate (2.5 to 7.5 mm/hr), heavy (>7.5 mm/hr)

Snow {
- Ice crystals in the form of flakes, dust, corn, etc
- typical density 0.05 − 0.15 g/cm³

Drizzle {
- liquid , <0.5 mm
- fine floating droplets , rate 1 mm/hr

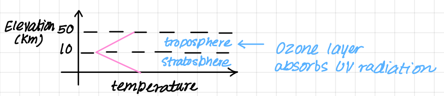
Glaze
- Freezing rain, result of rain on cold ground (typ. snow covered at winter end)

Sleet
- Frozen raindrops , while falling thro' air at subfreezing temp.

Hail
- Intense rain with lumps / pellets of ice, often > 8 mm, + thunderstorm .

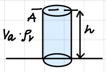
WATER VAPOUR

(Chow, 3.2, Water Vapor, Pg 56 − 64)



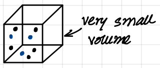
← Ozone layer absorbs UV radiation

$Ah = V_a \cdot \rho_v$



Precipitable water : mass of water present in a static air column

Properties of Water Vapour



- Density of air (ρ_a) = $\dfrac{m_a}{V_a}$
- Density of water vapour (ρ_v) = $\dfrac{m_v}{V_a}$
- specific humidity , $\boxed{q = \dfrac{\rho_v}{\rho_a}}$ (ratio of densities of water vapor and moist air) → Kg/Kg − unitless
  mass of water vabor per unit mass of moist air.
- Ideal Gas law,

$$pV_a = n_a RT \Rightarrow p = \frac{m_a}{V_a} \cdot \underbrace{\frac{R}{MM_a}}_{R_a} T = \rho_a \cdot \frac{R}{MM_a} \cdot T = \rho_a R_a T$$

T in kelvin, R = gas constant = 8.314 J/K·mol

- e = vapour pressure,

$$eV_a = n_v RT \Rightarrow e = \frac{m_v}{V_a} \cdot \underbrace{\frac{R}{MM_v}}_{R_v} T = \rho_v \cdot \frac{R}{MM_v} \cdot T = \rho_v R_v T$$

$$q = \frac{\rho_v}{\rho_a} = \frac{\frac{e\,MM_v}{RT}}{\frac{p\,MM_a}{RT}} = \frac{e\,MM_v}{p\ MM_a} = \frac{e}{p} \times \frac{18}{29} = 0.62\frac{e}{p} \Rightarrow \boxed{0.62\frac{e}{p} = q}$$

0.622e/p = q

- e_s = saturation vapor pressure

For a given temp., there is a maximum moisture content the air can hold, and the corresponding vapor pressure is called saturation vapor pressure.

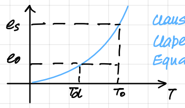
$$\boxed{e_s = 611\exp\left(\frac{17.27\,T}{237.3 + T}\right)}$$

e_s → Pa (N/m²), T → T °C

For a given temperature,

$$\boxed{\Delta = \frac{de_s}{dT} = \frac{4098 e_s}{(237.3 + T)^2}}$$

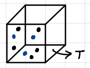
Δ → Pa/°C, T → T °C



Claussius Claperon Equation

- Relative Humidity = $\dfrac{e}{e_s} \times 100$ → e: actual vapor pressure, e_s: saturation vapor pressure

used in weather apps.
specific humidity is different from relative humidity.



- Dew point temperature (Td)

The temperature at which air would just become saturated at a given specific humidity. At this point, saturation vapor pressure = actual vapor pressure ($e_s = e$). For a given air with a given temp., when we reduce the temp. the ability to hold moisture ↓es, air will become more saturated. In winter water starts converting to dew even though . . . . Humidity ↑, more vapor pressure , temperature ↓es.

Example

Air pressure = 100 KPa, Air sample with temperature T = 20°C , Td = 16°C .

a) Calculate the saturation vapor pressure
b) " " actual " " at 20°C .
c) relative humidity
d) specific humidity
e) air density

Soln

a) $e_s = 611\exp\left(\dfrac{17.27 \times 20}{237.3 + 20}\right) = 2339$ Pa or 2.339 KPa

b) e is calculated by same eqn just by substituting Td (dew pt. temp)
$e = 611\exp\left(\dfrac{17.27 \times 16}{237.3 + 16}\right) = 1819$ Pa or 1.819 KPa ↳ at this pt. $e = e_s$

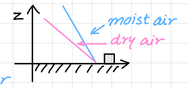
c) $R_h = \dfrac{e}{e_s} \times 100 = 78\%$

d) $q = 0.622\dfrac{e}{p} = 0.622 \times \dfrac{1819}{100 \times 10^3} = 0.0113$ Kg water / Kg moist air

e) air density ρ_a is calculated from ideal gas eqn

$\rho_a = \dfrac{p}{R_a T} = \dfrac{100 \times 10^3}{289 \times 293}$ ↳ T = 20°C + 273 = 293 K

$\boxed{\begin{array}{l} R_a = R_d(1 + 0.608\,q) \\ = 287(1 + 0.608q) \\ \text{J/Kg·K}\end{array}}$



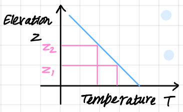
Dry air
Adiabatic − temperature change in
lapse rate − the absence of external energy change = 10°C/Km
moist air lapse rate = 5°C/Km

Revise

- specific gravity q $\dfrac{\rho_v}{\rho_a} = 0.62\dfrac{e}{p}$
- Saturation vapor pressure (es)
  - └ empirical relation
- Dew point temp. (Td)
  Relative humidity $R_h = \dfrac{e}{e_s} \times 100\%$

8 Aug

Water Vapour in a static Atmospheric column



- $-\frac{dT}{dz} = \alpha$ (lapse rate, α) $\longrightarrow$ $dz = -\frac{dT}{\alpha}$

$-dT = \alpha dz$
$-(T_2 - T_1) = \alpha(z_2 - z_1)$
$T_2 = T_1 - \alpha(z_2 - z_1)$
↳ Rem.

- $pV = nRT$
$p = \rho_a \frac{R}{M_a} T = \rho_a R_a T \Rightarrow \rho_a = \frac{p}{R_a T}$ ($\frac{R}{M_a} \to R_a$)

$$\boxed{T_2 = T_1 - \alpha(z_2 - z_1)}$$

- $\frac{dp}{dz} = -\rho_a g$

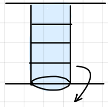
$-dp = \rho_a g dz$ = change in pressure

$dp = \rho_a g \frac{dT}{\alpha} = \frac{p g dT}{R_a T \alpha} \Rightarrow \frac{dp}{p} = \frac{g}{R_a \alpha} \cdot \frac{dT}{T}$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \frac{g}{R_a\alpha} \int_{T_1}^{T_2} \frac{dT}{T} \Rightarrow \ln\left(\frac{p_2}{p_1}\right) = \frac{g}{R_a\alpha} \ln\left(\frac{T_2}{T_1}\right)$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{g/(R_a\alpha)}}$$ T in Kelvin

Precipitable water The amount of moisture in an atmospheric column.



$q_w = \int_{z_1}^{z_2} \rho_v A dz$ $\rho_v = q\rho_a$
$= \int_{z_1}^{z_2} q \rho_a A dz$ A = constt.
↳ mass of precipitable water

one problem in quiz

quiz question
↓
lapse rate - given
find ρ_a = ?
e - find from dew pt. temp. (prev. class)

In each discretized column, using average

$$\boxed{q_w = \bar{q}\bar{\rho}_a A dz}$$ $\bar{q} = \frac{q_1 + q_2}{2}$ $\bar{\rho}_a = \frac{\rho_{a1} + \rho_{a2}}{2}$

Imp question for exams

★ **Example** calculate precipitable water in a saturated air column 10 Km high above 1 m² of ground surface. surface pressure = 101·3 KPa, surface air temp 30°C. lapse rate = 6·5°C/Km.

Sol^n take Δz = 2Km or 2000 m $T_1 = 30°C$ or 303 K
$z_1 = 0$ m, $z_2 = 2000$ m $\alpha = 6.5°C/Km = 0.0065°C/m$

z	T°C	T(K)	p (KPa)	ρ_a (Kg/m³)	e (KPa)	q_v	$\bar{q}_v$	$\bar{\rho}_a$	q_w (Kg)
0	30	303	101·3 KPa	1·16	4·24	0·0261	0·0205	1·07	43·7
2	17	290	80·4	0·97	1·94	0·0150	0·0115	0·88	20·2
4	4	277	63·2	0·79	0·81	0·0080	0·0060	0·72	8·6
6	-9	264	49·1	0·65	0·31	0·0039	0·0028	0·59	3·3
8	-22	251	37·6	0·52	0·10	0·0017	0·0012	0·47	1·1
10	-35	238	28·5	0·42	0·03	0·0007			
									77

Thus, total mass of precipitable water = 77 kg

$T_2 = T_1 - \alpha(z_2 - z_1)$
↳ $T_2 = 17°C$ or 290 K.

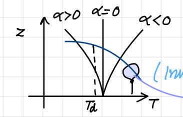
$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{g/(\alpha R_a)}$ (K; 9·8 m/s²; 0·0065; 287) = 80·4 KPa

$\rho_a = \frac{p}{R_a T} = \frac{101.3 \times 10^3}{287 \times 303} = 1.16$ Kg/m²

$e = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$ Pa, °C

$q_v = 0.622\, e/p$

$q_w = \bar{q}_v \bar{\rho}_a A \Delta z = 43.7$ Kg (1m², 2000)



(Inversion)

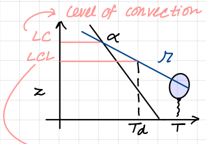
$-\frac{dT}{dz} = \alpha$ (environmental lapse rate)

→ Air inside not exchanging energy with surrounding.

Rate at which air inside the baloon cools (purely due to expansion) is called adiabatic lapse rate → This lapse rate will always be the same.

$\Gamma_{saturated} = 5°C/Km$
$\Gamma_{dry, unsaturated} = 10°C/Km$
↳ $e < e_s$

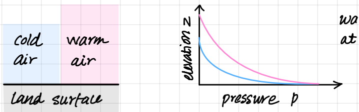
Level of convection (LC): maximum height acquired by the air parcel where air parcel temp. = surrounding temp.
If dew point temp. is achieved before level of convection, then it'll be condensed.



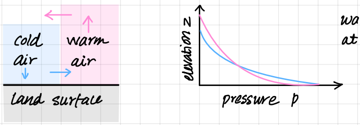
- Cloud formation if LC > LCL
- No Cloud formation if LC < LCL

Steps involved in precipitation:
- Cooling of air to its dew point by lifting
- Condensation of water vapor
- Growth of water droplets by coalescence and collision
- Precipitation of water droplets when they become sufficiently heavy.

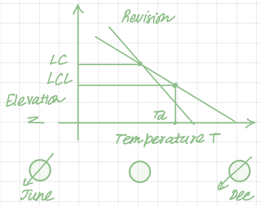
Pressure systems



warmer air has higher pressure at higher levels



warmer air has higher pressure at higher levels



10 Aug

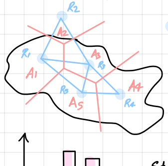
- **Rainfall Measurement**
  - hyetographs
  - cummulative hyetographs : mass curves

Need for spatial averaging of rainfall
$\Delta S = (P + G_{in}) - (E + T + G_{out} + Q)$

Rain Gauge
(also called hyetometer)
- Tipping bucket type rain gauge
- US standard rain gauge

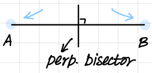
can also find distribution of rainfall

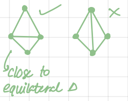
- ★ **Thiessen Polygon Method** → one question in exam ★

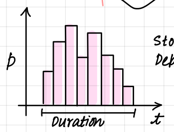


- Average precipitation

$$\boxed{\bar{p} = \frac{1}{A}\sum_{i=1}^{n} p_i A_i \qquad A = \sum_{i=1}^{n} A_i}$$







$$\boxed{\text{Intensity} = \frac{\text{Storm Depth}}{\text{Duration}}}$$

- Storm depth
- storm duration
- storm intensity = $\frac{\text{depth}}{\text{duration}}$
- frequency

- **Project life**

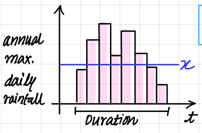
Portable Maximum Precipitation (PMP) = $P_{obs}^{max} \times \left(\frac{m_v^{max}}{m_{v,obs}}\right)$

P_{obs}^{max} – Maximum rain observed
$m_{v,obs}$ – Precipitable water on the day of P_{obs}^{max}.
m_v^{max} – maximum observed precipitable water.

- **Design life**

Return Period – average no. of years in which rainfall of x is exceeded.
RT(x) (or of magnitude $X > x$ is expected to occur)

$$\boxed{RT(x) = \frac{1}{1 - P_r(X \leq x)}}$$



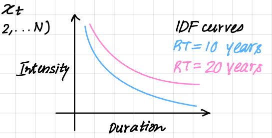
x raintall for which return period is to be calculated.

$P_r(X \leq x) = \frac{n(X \leq x)}{N}$ = $\frac{1}{1 - 36/40} = \frac{1}{4/40} = 10$ (for some data)
N ↑ length of data

17 Aug

Intensity - Duration - Frequency (IDF) analysis

1. Choose a duration & a return period.
2. Annual maximum rainfall in duration D → x_t
3. Arrange x_t in asunding order → rank i (= 1, 2, . . N)
4. $P_r(X \leq n) = \frac{i}{N}$
5. Return Period $RT(x) = \frac{1}{1 - P_r(X \leq x)}$



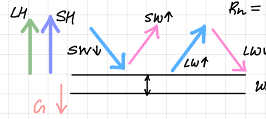
Evaporation

(Chow, 3·5, Pg 80)

- **Potential Evaporation (Ep):** Rate of evaporation under unlimited supply of water. water bodies like lakes, oceans, saturated soil.

- **Energy balance method:**

Net Radiation
$R_n = SW_\downarrow - SW_\uparrow + LW_\downarrow - LW_\uparrow$



LH – latent heat – energy consumed in phase change from liquid to vapor.
SH – sensible heat – energy transfer due to temperature difference
G – Ground heat flux

H – energy stored in the top layer of water/soil.
$\frac{dH}{dt}$ = Inflow of energy – outflow of energy

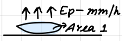
$\frac{dH}{dt} = R_n - LH - SH - G$

$LH + SH = \underbrace{R_n - G - \overbrace{\frac{dH}{dt}}^{\text{small usually}}}_{Q_n}$

$$\boxed{\frac{dH}{dt} = R_n - G - SH - LH}$$
Net radiation, ground, sensible heat, latent heat

$Q_n = LH + SH$
↳ R_n in many books (mostly equal)

Maximum rate of evaporation when $Q_n = LH$. (or R_n)



$LH = \lambda \times$ mass of water evaporated per unit time $= \rho_w \lambda E_p$ ($A \times E_p \times \rho_w$)
↑
latent heat of vaporization
Energy consumed for converting 1 Kg of water from liquid to vapor.
$\lambda = 2.5 \times 10^6$ J/Kg.

$\rho_w \lambda E_p = Q_n$ $\boxed{E_p = \frac{Q_n}{\rho_w \lambda}}$ ↳ energy

Revision

$\frac{dH}{dt} = R_n - G - SH - LH \Rightarrow SH + LH = R_n - G - \frac{dH}{dt} = Q_n$
($\frac{dH}{dt} \to 0$)
$SH + LH = Q_n$ $\rho_w \lambda E_p = Q_n$ $E_p = \frac{Q_n}{\rho_w \lambda}$

Energy Balance Method
λ – latent heat of vaporisation

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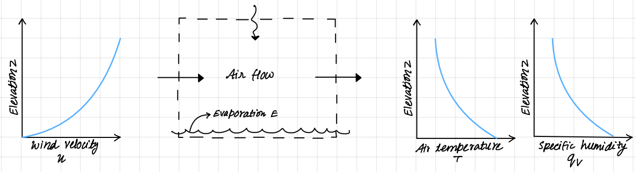
Aerodynamic Method : Effect of Wind and Humidity

24 Aug



$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz} = -\rho_a K_w \frac{q_{v1}-q_{v2}}{z_1-z_2}$ → kg/s

K_w : vapor eddy diffusivity

$\tau = \rho_a K_m \frac{du}{dz} = \rho_a K_m \frac{u_1-u_2}{z_1-z_2}$

K_m : momentum diffusivity

$E_{p,aero} = \frac{\dot{m}_v}{\rho_v} = \frac{K_w K^2 \rho_a (q_{v1}-q_{v2})(u_2-u_1)}{\rho_v K_m [\ln(z_2/z_1)]^2}$ → m^3/s

K: von Karman constant = 0.4

Thornthwaite - Holzman equation

Rem ☆

$$\boxed{E_{p,aero} = K_E (es_1 - e_2) u_2}$$

es_1 = saturation vapor pressure at the water surface
e_2 = vapor pressure at a fixed height from the surface (2m)
u_2 = wind velocity at a fixed height from the surface (2m)

$K_E = \frac{0.62 K_w K^2 \rho_a}{\rho_v K_m p_a}$

$q = 0.62\, e/p_a$
$u_1 = 0$ (zero vel. at surface)

Rate of evaporation $\propto (q_{v1} - q_{v2})$ ——— 2
$\propto (u_2 - u_1)$ ——— 1

This is known as estimation of evaporation by aerodynamic method.

Combining Energy Balance and Aerodynamics : PENMAN EQUATION

imp ☆

$$\boxed{E_p = \frac{Q_n}{\rho_w \lambda} \frac{\Delta}{\Delta+\gamma} + K_E u_2 (e_{s2}-e_2) \frac{\gamma}{\Delta+\gamma}}$$

Penman's equation states that the potential evaporation is the weighted average of contributions of solar radiation and wind advection.

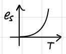
$$E_p = \frac{Q_n}{\rho_w \lambda} \underbrace{\frac{\Delta}{\Delta+\gamma}}_{\text{Weight for energy-based evaporation}} + K_E u_2 (e_{s2}-e_2) \underbrace{\frac{\gamma}{\Delta+\gamma}}_{\text{Weight for wind advection based evaporation}}$$

$\Delta = \frac{4098 e_s}{(237.3+T)^2}$ ≠ constant

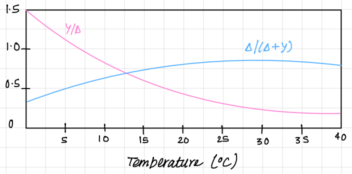
$\gamma = \frac{C_p K_h p_a}{0.62 K_w \lambda} = 67\,Pa/K$ = constant

es_2 = saturation vapor pressure at a fixed height from the surface (2m)

$\Delta = \frac{de_s}{dT}$



γ = psychrometric constant



$\frac{\Delta}{\Delta+\gamma}$ ↑es with temp.

Thus, as temp. ↑es, weightage of energy compoment ↑es i.e. at higher temperature, energy has a more dominating effect on evaporation. Whereas, at lower temperature, humidity has a control over evaporation.

Standard National Weather Service evaporation pan having a diameter of 4 ft and height of 10 inch

PAN EVAPORATION METHOD

$\Delta S = \text{Inflow} - \text{outflow} = P - E$

$S_2 - S_1 = P - E$

↳ $E = P + S_1 - S_2$

$$\boxed{E_{pan} = S_1 - S_2 + P}$$

P → Rainfall during measurement period.

S_2 → storage at the end of measurement period.

S_1 → storage at beginning

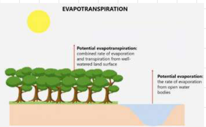
$$\boxed{E_{lake} = E_{pan} \times K_{pan}}$$

K_{pan} – pan coefficient (<1)
The average value of K_{pan} is 0.7

Evapotranspiration

(Chow, 3.6, Pg 91)

Evaporation is sum of evaporation (from water bodies) and transpiration.



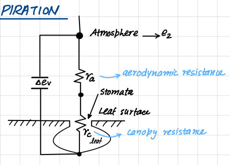
To estimate evapotranspiration, we use a method called penman - monteith method.

PENMAN - MONTEITH METHOD FOR POTENTIAL EVAPOTRANSPIRATION

$E_p = \frac{0.62 \rho_a}{p_a} \left(\frac{e_{s1} - e_2}{r_c + r_a} \right)$

$$\boxed{E_p = \frac{\frac{\Delta R_n}{\rho \lambda} + \frac{\rho_a C_p}{\lambda r_a}(e_{s2} - e_2)}{\Delta + \gamma \left(\frac{r_c + r_a}{r_a} \right)}}$$

Penman-Monteith Equation



Penman Monteith eqⁿ is used to estimate the crop water requirements in irrigation scheduling.

Canopy resistance is how easily the vegetation is transpiring water. If it is closing the stomata, the resistance will be higher. If you've more no. of leaves in a vegetation, the resistance will be lower. Also it depends on stage of plant growth.
Initial stage – high resistance
Mature stage – low resistance (req. more water)

canopy resistance depends on
- No. of leaves / leaf density
- Stomatal opening
- Stage of growth

es_1 – sat. vapor pressure at leaf surface
e_2 – vapor pressure at a fixed height above surface (2m)

aerodynamic resistance represents effect of wind. more wind, more r_a.

- The aerodynamic resistance can be estimated from wind velocity measurements

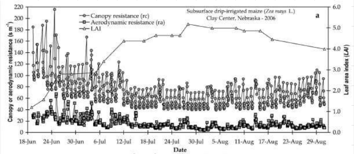
$$\boxed{r_a = \frac{\ln\left(\frac{z_m - d}{z_{om}}\right) \ln\left(\frac{z_h - d}{z_{oh}}\right)}{k^2 u_z}}$$

z_m : height of wind measurement (3m)
z_h : height of humidity measurement (2m)
d : 0.67 × plant height (h)
z_{om} : 0.123h
z_{oh} : 0.10 z_{om}

- However the estimation of surface resistance is very difficult as it not only depends on the physiological characteristics of plants but also meteorological variables.

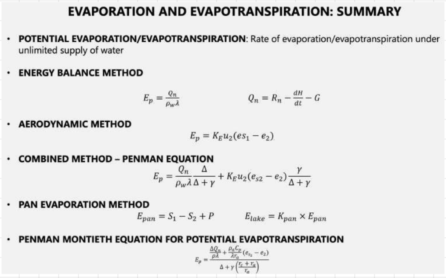
$$\boxed{E_p = \frac{\Delta R_{ne} + \frac{\rho_a C_p}{\lambda r_a}(e_{s2} - e_2)}{\Delta + \gamma \left(\frac{r_c + r_a}{r_a} \right)}}$$

r_c is estimated from observed E_p measurements through calibration.



Irmak and Mutiibwa (2010)

- Actual evaporation ≤ Potential evaporation



EVAPORATION AND EVAPOTRANSPIRATION: SUMMARY

- **POTENTIAL EVAPORATION/EVAPOTRANSPIRATION**: Rate of evaporation/evapotranspiration under unlimited supply of water
- **ENERGY BALANCE METHOD**

$$E_p = \frac{Q_n}{\rho_w \lambda} \qquad Q_n = R_n - \frac{dH}{dt} - G$$

- **AERODYNAMIC METHOD**

$$E_p = K_E u_2 (es_1 - e_2)$$

- **COMBINED METHOD – PENMAN EQUATION**

$$E_p = \frac{Q_n}{\rho_w \lambda} \frac{\Delta}{\Delta+\gamma} + K_E u_2 (e_{s2} - e_2) \frac{\gamma}{\Delta+\gamma}$$

- **PAN EVAPORATION METHOD**

$$E_{pan} = S_1 - S_2 + P \qquad E_{lake} = K_{pan} \times E_{pan}$$

- **PENMAN MONTIETH EQUATION FOR POTENTIAL EVAPOTRANSPIRATION**

$$E_p = \frac{\frac{\Delta Q_n}{\rho \lambda} + \frac{\rho_a C_p}{\lambda r_a}(e_{s_2} - e_2)}{\Delta + \gamma \left(\frac{r_c + r_a}{r_a} \right)}$$

Example Evaporation rate = ? by energy balance method — open water surface
R_n = net radiation = 200 W/m²   Air temp = 25°C
Assume no sensible heat or ground flux.

Solⁿ

$E_r = \frac{Q_n}{\rho_w l_v} \approx \frac{R_n}{\rho_w l_v} = \frac{200}{\underbrace{2441 \times 10^3}_{l_v = 2500 - 2.36 \times \underset{T^\circ C}{25}} \times \underset{\rho_w}{997}} = 8.22 \times 10^{-8}\, m/s \approx 7.10\, mm/day$

Example E_p = ? Aerodynamic method, open water surface, air temp = 25°C, R_h = 40%.
p_a = 101.3 kPa u_2 = wind speed = 3 m/s z_0 = 0.03 cm z_2 = 2 m

Solⁿ: $B = \frac{0.622 K^2 \rho_a u_2}{p \rho_w [\ln(z_2/z_0)]^2} = \frac{0.622 \times 0.4^2 \times 1.19 \times 3}{101.3 \times 10^3 \times 997 [\ln(2/3\times10^{-4})]^2} = 4.54 \times 10^{-11}\, m/Pa.s$

$E_a = B(e_{as} - e_a) = B(e_{as} - R_h e_{as}) = B(3167 - 0.4 \times 3167) = 8.62 \times 10^{-8}\, m/s$
$= 7.45\, mm/day$

Example E_p = ? combined method open water surface, air temp 25°C, R_h = 0.4
u_2 = 3 m/s, z_2 = 2m, p = 101.3

Solⁿ: $\gamma = \frac{C_p K_h p}{0.62 K_w l_v} \approx 67\, Pa/°C$ $\frac{\Delta}{\Delta+\gamma}$ $\frac{\gamma}{\Delta+\gamma}$

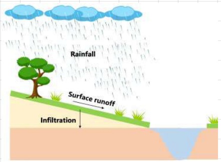
$\Delta = \frac{4098 e_s}{\quad}$ ~ $e_s = e_{as} = 3167$ for 25°C

$\therefore E = \frac{\Delta}{\Delta+\gamma} E_r + \frac{\gamma}{\Delta+\gamma} E_a = 7.2\, mm/day$

Infiltration

29 Aug

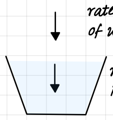
- What is infiltration?
  Infiltration is the process of water penetrating from the ground surface into the soil.

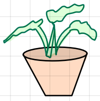


- Rainfall < infiltration / surface runoff

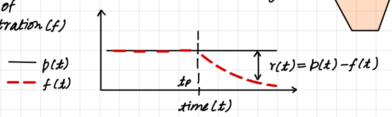
- Applications < streamflow forecasting / urban drainage systems and green infrastructure design / Irrigation systems

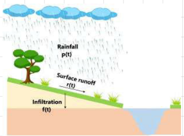
- An example from daily life — watering of flower pots/green plants.



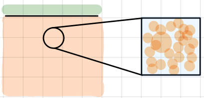


t_p = time of ponding
$r(t)$ = rate of ponding





- ## Soil Porosity and Soil Moisture

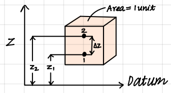


Soil Porosity $= \eta = \frac{\text{volume of voids}}{\text{total volume}}$

Soil moisture $= \theta = \frac{\text{volume of water } (V_w)}{\text{total volume } (V_T)}$ ↳ solid + void volume

$0 \leq \theta \leq \eta$
$V_W = \theta \times V_T$

- ## Darcy's law



$$\boxed{q = -K\frac{\partial h}{\partial z}}$$

q: $\frac{Q}{A}$

K: coefficient of permeability / hydraulic conductivity [L/T]

Why −ve sign? flow goes from higher energy state to lower ($q=-f$)

$$h = \frac{p}{\rho g} + z + \cancelto{0}{\frac{v^2}{2g}} \leftarrow \text{energy per unit weight}$$
$$= \frac{p}{\rho g} + z$$

$q>0$ — flow takes place in +ve z dir^n (upwards)
$q<0$ — flow takes place in −ve z dir^n (downwards)

$$\boxed{q = -K\frac{\partial h}{\partial z}}$$

Saturated soil

$q = -K_s\frac{\partial h}{\partial z}$ — K_s: saturated hydraulic conductance

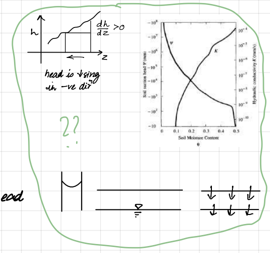
$h = z + \frac{p}{\rho g}$

Unsaturated soil

$q = -K(t)\left[\frac{\partial h(\theta)}{\partial z}\right]$

$K \neq$ constant

$= K(\theta)\frac{\partial}{\partial z}(z + \psi(\theta))$ — ψ: suction head



Example: What is the flow per unit area and in which dir^n? $\psi = 0$ for saturated.

$z_1 = 1.2m$ $\psi_1 = -2.2m$ $\Delta z = 1m$
$z_2 = 0.2m$ $\psi_2 = -0.5m$ $K = 4\times10^{-4} m/day$

d - diameter of soil particles

Soln:

$Re = \frac{\rho v d}{\mu} \leqslant 1$ $q = -K\frac{\partial h}{\partial z}$

$h_1 = \psi_1 + z_1 = 12 - 22 = -1$
$h_2 = \psi_2 + z_2 = 0.2 - 0.5 = -0.3$

$$q = -K\frac{\partial h}{\partial z} = -4\times10^{-4}\frac{m}{day} \times \frac{-0.3-(-1)}{0.2-1.2} = 2.8\times10^{-4}\ m/day$$

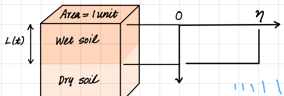
$$q = Q\cdot A \Rightarrow Q = \frac{q}{\underset{1unit}{A}} = 2.8\times10^{-4}\ /m/day$$

See from other's notes late in class!

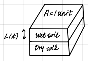
2. q_{actual} =

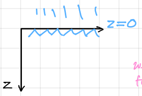
- ## Green Ampt Equation

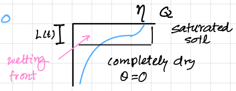
31 Aug



Assumption 1: soil moisture profile







Therefore at any time t,
total volume of water infiltrated $F(t) = V_T \times \eta = L(t)\times A\times\eta = L(t)\times1\times\eta$ (V_T: volume of soil)
$F(t) = L(t)\times\eta$

$F(t)$: cumulative infiltration; $L(t)$: ≠ constt, changing w.r.t. time

- Potential Infiltration Capacity f_* : The maximum rate at which water is entering the soil.

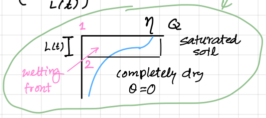
Assumption 2: uniform flow rate in the wet soil.
Assumption 3: The potential infiltration rate is equal to the rate of flow b/w points 1 and 2. $f_* = q_{12}$

$$q = -K\frac{\partial h}{\partial z} = -K_s\frac{\partial h}{\partial z} = -K_s\frac{(-L(t)-\psi_0)}{-L(t)-0} = -K_s\left(1+\frac{\psi_0}{L(t)}\right)$$??

$\frac{\partial h}{\partial z} = \frac{h_2 - h_1}{z_2 - z_1}$

$h_1 = z_1 + \cancelto{0}{\frac{p_1}{\rho g}}$

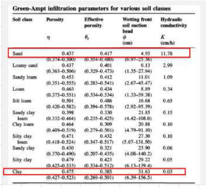
$h_2 = z_2 + \psi_2 = -L(t) - \psi_0$ ← suction head at the wetting front



(unsaturated soil—suction always)

$$\boxed{f_*(t) = q_{12} = -K_s\left(1 + \frac{\psi_0\eta}{F(t)}\right)}$$ Green Ampt Equation

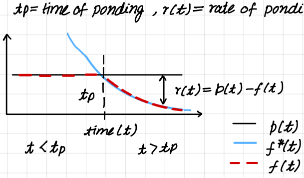
Green Ampt Parameters K_s, η, ψ_0



Green-Ampt infiltration parameters for various soil classes

Soil class	Porosity η	Effective porosity θ_e	Wetting front soil suction head ψ (cm)	Hydraulic conductivity K (cm/h)
Sand	0.437 (0.374–0.500)	0.417 (0.354–0.480)	4.95 (0.97–25.36)	11.78
Loamy sand	0.437 (0.363–0.506)	0.401 (0.329–0.473)	6.13 (1.35–27.94)	2.99
Sandy loam	0.453 (0.351–0.555)	0.412 (0.283–0.541)	11.01 (2.67–45.47)	1.09
Loam	0.463 (0.375–0.551)	0.434 (0.334–0.534)	8.89 (1.33–59.38)	0.34
Silt loam	0.501 (0.420–0.582)	0.486 (0.394–0.578)	16.68 (2.92–95.39)	0.65
Sandy clay loam	0.398 (0.332–0.464)	0.330 (0.235–0.425)	21.85 (4.42–108.0)	0.15
Clay loam	0.464 (0.409–0.519)	0.309 (0.279–0.501)	20.88 (4.79–91.10)	0.10
Silty clay loam	0.471 (0.418–0.524)	0.432 (0.347–0.517)	27.30 (5.67–131.50)	0.10
Sandy clay	0.430 (0.370–0.490)	0.321 (0.207–0.435)	23.90 (4.08–140.2)	0.06
Silty clay	0.479 (0.425–0.533)	0.423 (0.334–0.512)	29.22 (6.13–139.4)	0.05
Clay	0.475 (0.427–0.523)	0.385 (0.269–0.501)	31.63 (6.39–156.5)	0.03

1. Rate of rainfall — p
2. Potential Infiltration rate — f_* ⇝ not actual
3. Actual Infiltration rate — f
   t_p = time of ponding, $r(t)$ = rate of ponding



Initially it is driven by suction then friction and gravity.

- At small times, $\frac{\psi_0\eta}{F(t)}$ is very large and hence suction force dominate at short time-scales.

- At large times, $\frac{\psi_0\eta}{F(t)}$ is very small and hence friction and gravity forces are more important at longer time-scales.

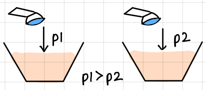
- When $p(t) < f_*(t)$, $f(t) = p(t)$
- When $p(t) > f_*(t)$, $f(t) = f_*(t)$

$\Big\}$ $f(t) = \min(p(t), f_*(t))$

$$F(t) = \int_0^t f(t)\,dt$$

Rem. ☆ V. Imp.

QUESTION At a given time t, the total volume of water in the two soils is the same $F_1(t) = F_2(t)$
Which soil sample will have a higher maximum allowable infiltration $f_*(t)$?



Soln :- $f_*(t)_1 > f_*(t)_2$

higher rainfall will have higher $f_*(t)$.

V.Imp.★

GREEN AMPT EQUATION : SOLVED EXAMPLE Imp★

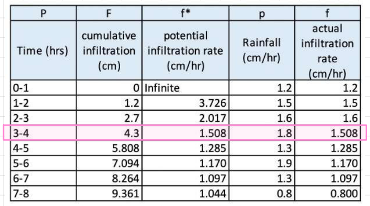
$\eta = 0.34$
$\psi_0 = 16.7$ cm
$K_s = 0.65$ cm/hr
The rainfall intensity variation is as follows:-

Time (hrs)	Rainfall Intensity (cm/hr)
0-1	1.2
1-2	1.5
2-3	1.6
3-4	1.8
4-5	1.3
5-6	1.9
6-7	1.3
7-8	0.8

Calculate the actual infiltration rate as a function of time using the Green Ampt equation.

Soln To find rate of infiltration in time interval $t_1 - t_2$.

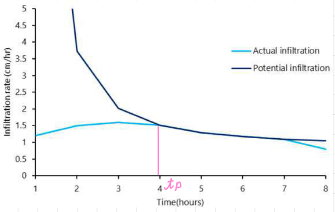
1. Calculate cumulative infiltration at a time $t_1 = F(t_1)$
2. Use Green Ampt equation to find potential infiltration rate
   $f_*[t_1,t_2] = K_s\left(1 + \frac{\psi_0\eta}{F(t_1)}\right)$
3. If rainfall during $t_1 - t_2$ is less than $f_*(t_1)$ then actual infiltration rate $f[t_1,t_2]$ = precipitation $p[t_1,t_2]$
   If rainfall during $t_1 - t_2$ is greater than $f_*(t_1)$ then actual infiltration rate $f[t_1,t_2]$ = potential infiltration rate $f_*[t_1,t_2]$
4. Use the actual infiltration rate to compute the cumulative infiltration at the end of time t_2 $F(t_2) = F(t_1) + f[t_1,t_2]\times(t_2 - t_1)$
5. Repeat the steps 1-4 for next time steps.



P	F	f*	p	f
Time (hrs)	cumulative infiltration (cm)	potential infiltration rate (cm/hr)	Rainfall (cm/hr)	actual infiltration rate (cm/hr)
0-1	0	Infinite	1.2	1.2
1-2	1.2	3.726	1.5	1.5
2-3	2.7	2.017	1.6	1.6
3-4	4.3	1.508	1.8	1.508
4-5	5.808	1.285	1.3	1.285
5-6	7.094	1.170	1.9	1.170
6-7	8.264	1.097	1.3	1.097
7-8	9.361	1.044	0.8	0.800

→ t_p (ponding time)
time at which $p > f_*$

$\eta = 0.34$
$\psi_0 = 16.7$ cm
$K_s = 0.65$ cm/hr

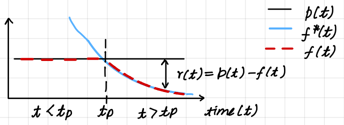


PONDING TIME (tp)

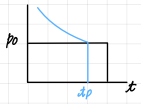
5 Sept

The ponding time tp, is the elapsed time b/w the time rainfall begins and the time water begins to pond on the soil surface.

Rate of rainfall — p
Potential Infiltration rate — f_*
Actual Infiltration rate — f
tp = time of ponding
r(t) = rate of ponding



- $t < t_p$ — rainfall intensity < potential infiltration rate
  — soil surface unsaturated.
- $t = t_p$ — begins when rainfall intensity > potential infiltration rate.
  — soil surface saturated.
- $t > t_p$ — saturated zone deepens into soil and overland flow occurs from the ponded water.



$F(t_p) = p_o t_p$

$$f_* = K_s\left(1 + \frac{\psi_o \eta}{F}\right)$$

$$p_o = K_s\left(1 + \frac{\psi_o \eta}{F(t_p)}\right)$$

$$p_o = K_s\left(1 + \frac{\psi_o \eta}{p_o t_p}\right)$$

$$p_o^2 t_p = K_s(p_o t_p + \psi_o \eta)$$

$$t_p[p_o^2 - p_o K_s] = K_s \psi_o \eta$$

$$\boxed{t_p = \frac{K_s \psi_o \eta}{p_o(p_o - K_s)}}$$

Ponding time under the constant rainfall intensity using the Green Ampt equation.

Ven Te Chow ↷ ☆

For Exams → Practice solved examples from Ven Te Chow book

Example: Find tp and depth of water infiltrated at ponding for a silt loam soil of 30% initial eff. saturation, subject to rainfall intensities of
a) 1cm/h
b) 5cm/h

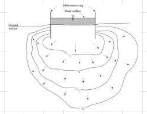
Soln $t_p = \frac{K_s \psi_o \eta}{p_o(p_o - K_s)}$ $K_s = 0.65$, $\psi \eta = 5.68$

a) $p_o = 1\text{cm/h} \longrightarrow t_p = 10.5\,h \longrightarrow F_p = p_o t_p = 10.5\text{cm}$
b) $p_o = 5\text{cm/h} \longrightarrow t_p = 0.17h$ (10 min) $\longrightarrow F_p = p_o t_p = 0.85\text{ cm}$

$f = \min[f_*, p]$ eg:- 3hr − ponding time
diff. p & f − ponding rate.

Measurement of Ponded Infiltration Rate

Ring Infiltrometer



Double Ring Infiltrometer

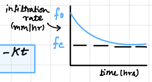
measurement is done only on this circular region (double rate)

Ponded Infiltration Rate Equations

$f_*(t)$ — infiltration capacity at time 't' from start of rainfall

Horton's equation

$$\boxed{f_*(t) = f_c + (f_o - f_c)e^{-Kt}}$$



f_o = initial infiltration rate at t = 0
f_c = ultimate infiltration capacity at t = tc
K = Horton's decay coefficient
t − time (in hours)

Green Ampt's equation

$$f_*(t) = K_s\left(1 + \frac{\psi_o \eta}{F(t)}\right)$$

ψ_o = capillary suction at wetting front
K = darcy's hydraulic conduc.
η = porosity

Philip's Equation

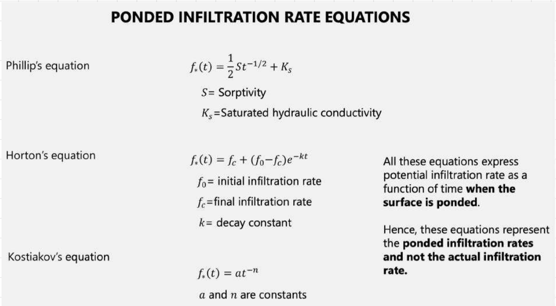
$$f_*(t) = \frac{1}{2} S t^{-1/2} + K_s$$

S = sorptivity = suction rate
K_s = darcy's hydraulic conductivity

Kostiakov's equation

$$f_*(t) = a t^{-n}$$

a & n − constants



PONDED INFILTRATION RATE EQUATIONS

Phillip's equation $f_*(t) = \frac{1}{2}St^{-1/2} + K_s$
S = Sorptivity
K_s = Saturated hydraulic conductivity

Horton's equation $f_*(t) = f_c + (f_0 - f_c)e^{-kt}$
f_0 = initial infiltration rate
f_c = final infiltration rate
k = decay constant

Kostiakov's equation $f_*(t) = at^{-n}$
a and n are constants

All these equations express potential infiltration rate as a function of time **when the surface is ponded**.

Hence, these equations represent the **ponded infiltration rates and not the actual infiltration rate.**

These are all for ponded state, right from t = 0. If the soil has not undergone ponding then we cannot use these equations in non-ponded state.

To use them in non-ponded state, we can use the following equation :-

$$f_* = -K_s\left(1 + \frac{\psi_o \eta}{F(t)}\right)$$ Time Condensation Form

Example: Philips eqn $f_*(t) = \frac{1}{2} S t^{-1/2} + K_s$ $K_s < \frac{S t^{-1/2}}{2}$ → eliminate time

Ponded state $f_*(t) = \frac{S t^{-1/2}}{2} = f(t)$

$$F(t) = \int_0^t f(t)\,dt = \frac{1}{2}\frac{S t^{1/2}}{1/2} = S t^{1/2}$$

$$t^{1/2} = \frac{F(t)}{S} \Rightarrow \boxed{f_*(t) = \frac{S \times S}{2F(t)} = \frac{S^2}{2F(t)}}$$

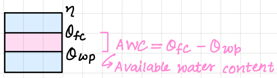
- **Field Capacity** θ_{fc} soil moisture at which flow in the soil is 0 (zero).

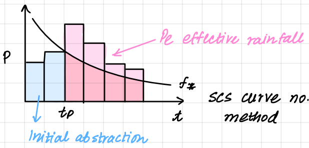
$$q = -K_s\frac{\partial h}{\partial z} = -K_s\left(1 + \frac{\partial \psi}{\partial z}\right) \qquad 1 + \frac{\partial \psi}{\partial z} > 0,\ \frac{\partial \psi}{\partial z} > -1$$

$$\frac{\partial \psi}{\partial z} = -1$$

- **wilting point** θ_{wp} soil moisture when $\psi < -14.7 \times 10^5$ Pa

Below wilting point, even if plant have water, it won't be able to use it.

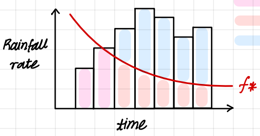




SCS curve no. method

SCS Method

12 Sept



I_a, Initial abstraction
F_a, Continuing abstraction, infiltration after rainfall
P_e, effective rainfall (runoff) — total depth of runoff produced during storm (not hourly)
f_* potential infiltration rate

SCS Curve Number Method
└ Soil Conservation Service
for computing depth of excess rainfall / direct runoff from a storm

Two points
1. two methods < graphical
   equations S = f(CN)
2. Both should be in inches (not SI unit)

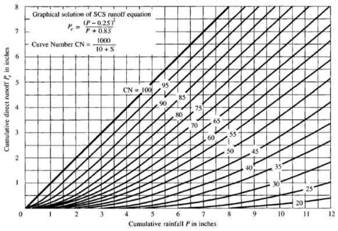
$$\boxed{P_e = \frac{(P - I_a)^2}{P - I_a + S}}$$

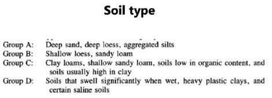
P_e : effective rainfall (runoff)
P : storm rainfall (inches)
S : parameter related to curve number (CN)
$S = \frac{1000}{CN} - 10$; $0 \leq CN \leq 1000$ → S (inches)

$$\boxed{I_a = 0.2S}$$

$$\boxed{P_e = \frac{(P - 0.2S)^2}{P + 0.8S}}$$

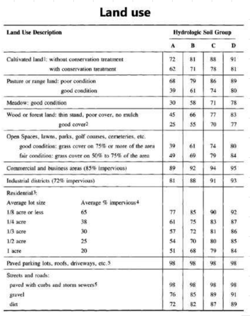
P − storm rainfall (inches)





Soil type

Group A: Deep sand, deep loess, aggregated silts
Group B: Shallow loess, sandy loam
Group C: Clay loams, shallow sandy loam, soils low in organic content, and soils usually high in clay
Group D: Soils that swell significantly when wet, heavy plastic clays, and certain saline soils



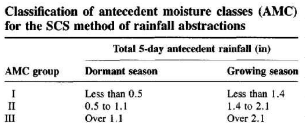
Land use

Land Use Description		Hydrologic Soil Group			
		A	B	C	D
Cultivated land1: without conservation treatment		72	81	88	91
with conservation treatment		62	71	78	81
Pasture or range land: poor condition		68	79	86	89
good condition		39	61	74	80
Meadow: good condition		30	58	71	78
Wood or forest land: thin stand, poor cover, no mulch		45	66	77	83
good cover2		25	55	70	77
Open Spaces, lawns, parks, golf courses, cemeteries, etc.					
good condition: grass cover on 75% or more of the area		39	61	74	80
fair condition: grass cover on 50% to 75% of the area		49	69	79	84
Commercial and business areas (85% impervious)		89	92	94	95
Industrial districts (72% impervious)		81	88	91	93
Residential3:					
Average lot size	Average % impervious4				
1/8 acre or less	65	77	85	90	92
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	86
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
Paved parking lots, roofs, driveways, etc.5		98	98	98	98
Streets and roads:					
paved with curbs and storm sewers5		98	98	98	98
gravel		76	85	89	91
dirt		72	82	87	89

Adjustment for antecedent conditions

$$\boxed{CN(I) = \frac{4.2CN(II)}{10 - 0.058CN(II)}}$$

$$\boxed{CN(III) = \frac{23CN(II)}{10 + 0.13CN(II)}}$$



Classification of antecedent moisture classes (AMC) for the SCS method of rainfall abstractions

	Total 5-day antecedent rainfall (in)	
AMC group	Dormant season	Growing season
I	Less than 0.5	Less than 1.4
II	0.5 to 1.1	1.4 to 2.1
III	Over 1.1	Over 2.1

curve number (CN) − based on land use land cover (LULC) and infiltration
high CN — less infiltration — more runoff (eff. rainfall)
CN for mixed LULC $= \left(\frac{\Sigma A_i * CN_i}{\Sigma A_i}\right)$ weighted average for mixture of LULC.

Example: compute runoff from 5 inches of rainfall on a 1000 acre watershed. Soil group − 50% soil B and 50% soil C interspersed throughout. AMC II is assumed. land use is 40% residential area i.e. 30% impervious
12% residential area i.e. 60% impervious

Soln: Weighted $CN = \frac{4038 + 4340}{100} = 83.8$

$$S = \frac{1000}{\underset{83.8}{CN}} - 10 = 1.93 \text{ in}$$

$$P_e = \frac{(P - 0.2S)^2}{P + 0.8S} = \frac{(5 - 0.2 \times 1.93)^2}{5 + 0.8 \times 1.93} = \underline{\underline{3.25 \text{ in}}}$$

If AMC III is assumed, the equivalent curve number will be

$$CN(III) = \frac{23\,CN(II)}{10 + 0.13\underset{83.8}{CN(II)}} = 92.3$$

$$S = \frac{1000}{CN(III)} - 10 = 0.83 \text{ in}$$

$$P_e = \frac{(\overset{5}{P} - 0.2S)^2}{P + 0.8\underset{0.83}{S}} = \underline{\underline{4.13 \text{ in}}}$$

change in runoff
$\Delta P_e = 4.13 - 3.25 = 0.88$ in
or 27% increase

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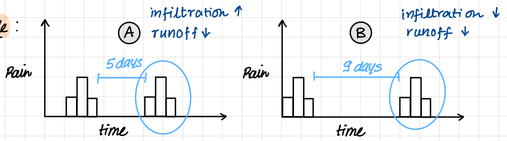
Example :



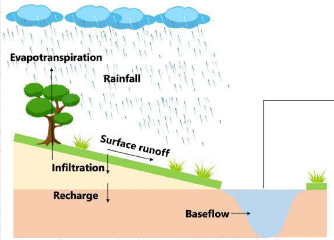
Here everything is same, just the no. of days are different.
Will the effective rainfall be equal or different?

Sol^n $P_e^A > P_e^B$, case A will have more runoff

Antecedent moisture condition ← account for previous rain condition here adjustments done

Summarize
- Rainfall
- Infiltration
- Surface Runoff
- Evapotranspiration
- Recharge ground water
- Baseflow — contribution of groundwater to flow in oceans / streams

HYDROLOGIC PROCESSES

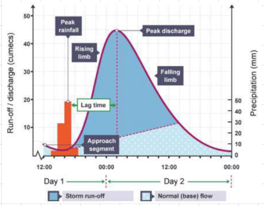


Streamflow = Surface runoff + Baseflow

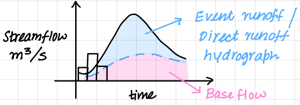
Surface runoff: Part of streamflow generated over the land surface ~ frequent

Baseflow: Part of streamflow contributed by the subsurface and groundwater ~ very slow

HYDROGRAPH



Flood Hydrograph

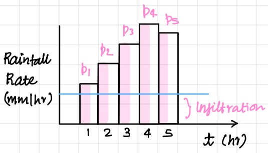


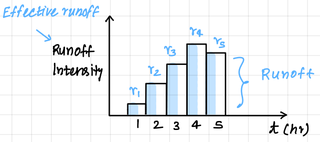
peak and runoff has a lag
as catchment size ↑es, more water storage capacity ...

- ordinates of DRH = ordinates of FH − Baseflow
- Area of DRH = volume runoff = (Σ of ordinates) $\times \Delta t$
- Volume of Runoff = Catchment Area × Runoff Depth
- Runoff depth = $\frac{\text{Vol. of runoff}}{CA} = \frac{\text{Area of DRH}}{CA}$

Effective Runoff Hyetograph (ERH)

Rainfall = Runoff + Infiltration

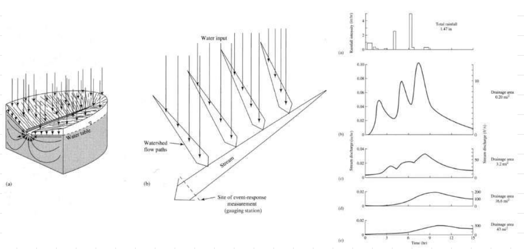




time	volume
1	r_1A_1
2	$r_2A_1 + r_1A_2$
3	$r_3A_1 + r_2A_2 + r_1A_3$
4	$r_4A_1 + r_3A_2 + r_2A_3 + r_1A_4$
5	$r_5A_1 + r_4A_2 + r_3A_3 + r_2A_4$

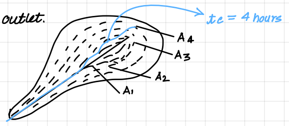
- Area of ERH = Total runoff depth
- Area of DRH = Volume of runoff
- Volume of runoff = CA × Runoff depth
- Area of DRH = CA × Runoff depth

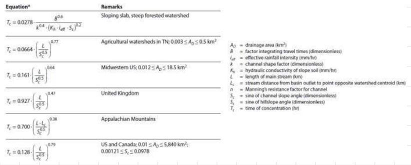
TRAVEL PATHWAYS OF WATER



Time of Concentration

Time taken to travel from the hydrologically most distant point of the watershed to the outlet.



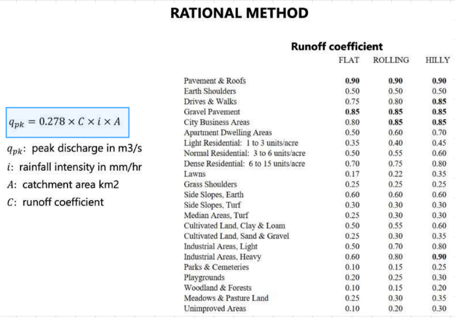


Equation*	Remarks
$T_c = 0.0278 \cdot \frac{B^{0.6}}{k^{0.4} \cdot (K_h \cdot i_{eff} \cdot S_s)^{0.2}}$	Sloping slab, steep forested watershed
$T_c = 0.0664 \cdot \left(\frac{L}{S_c^{0.5}}\right)^{0.77}$	Agricultural watersheds in TN; $0.003 \le A_D \le 0.5$ km²
$T_c = 0.161 \cdot \left(\frac{L}{S_c^{0.5}}\right)^{0.64}$	Midwestern US; $0.012 \le A_D \le 18.5$ km²
$T_c = 0.927 \cdot \left(\frac{L}{S_c^{0.5}}\right)^{0.47}$	United Kingdom
$T_c = 0.700 \cdot \left(\frac{L \cdot L_c}{S_c^{0.5}}\right)^{0.38}$	Appalachian Mountains
$T_c = 0.128 \cdot \left(\frac{L}{S_c^{0.5}}\right)^{0.79}$	US and Canada; $0.01 \le A_D \le 5{,}840$ km²; $0.00121 \le S_c \le 0.0978$

A_D = drainage area (km²)
B = factor integrating travel times (dimensionless)
i_{eff} = effective rainfall intensity (mm/hr)
k = channel shape factor (dimensionless)
K_h = hydraulic conductivity of slope soil (mm/hr)
L = length of main stream (km)
L_c = stream distance from basin outlet to point opposite watershed centroid (km)
n = Manning's resistance factor for channel
S_c = sine of channel slope angle (dimensionless)
S_s = sine of hillslope angle (dimensionless)
T_c = time of concentration (hr)

Rational Method

$q_{pk} = 0.278 \times C \times i \times A$ → for small watershed (around 1Km²)



RATIONAL METHOD

$q_{pk} = 0.278 \times C \times i \times A$

q_{pk}: peak discharge in m3/s
i: rainfall intensity in mm/hr
A: catchment area km2
C: runoff coefficient

Runoff coefficient

	FLAT	ROLLING	HILLY
Pavement & Roofs	**0.90**	**0.90**	**0.90**
Earth Shoulders	0.50	0.50	0.50
Drives & Walks	0.75	0.80	**0.85**
Gravel Pavement	**0.85**	**0.85**	**0.85**
City Business Areas	0.80	**0.85**	**0.85**
Apartment Dwelling Areas	0.50	0.60	0.70
Light Residential: 1 to 3 units/acre	0.35	0.40	0.45
Normal Residential: 3 to 6 units/acre	0.50	0.55	0.60
Dense Residential: 6 to 15 units/acre	0.70	0.75	0.80
Lawns	0.17	0.22	0.35
Grass Shoulders	0.25	0.25	0.25
Side Slopes, Earth	0.60	0.60	0.60
Side Slopes, Turf	0.30	0.30	0.30
Median Areas, Turf	0.25	0.30	0.30
Cultivated Land, Clay & Loam	0.50	0.55	0.60
Cultivated Land, Sand & Gravel	0.25	0.30	0.35
Industrial Areas, Light	0.50	0.70	0.80
Industrial Areas, Heavy	0.60	0.80	**0.90**
Parks & Cemeteries	0.10	0.15	0.25
Playgrounds	0.20	0.25	0.30
Woodland & Forests	0.10	0.15	0.20
Meadows & Pasture Land	0.25	0.30	0.35
Unimproved Areas	0.10	0.20	0.30

$C = \frac{\text{runoff}}{\text{rainfall}} \not<$

- Return Period = Duration ⟶ $R.T. = \frac{1}{P(\text{exceedence})}$ Annual rainfall peak discharge
- Choose Intensity that has this duration/return period
- Find peak discharge

For this duration = time of concentration
Intensity = constant

Exams — One question
find intensity from return period
— use it to find peak discharge.

(Midsem — mostly after quiz)

last ques. in quiz — IDF ⤳ return period
Duration ← I ↙

duration = time of conc^n
highest discharge.

- This works only for cumulative.
- It doesn't give hourly rainfall rate.

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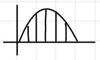
14 Sept

Unit Hydrograph

Note: It can be defined for 1mm/1cm/1in. In this course we use "1mm"

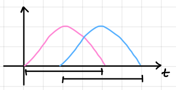
- It is a direct runoff hydrograph resulting from a unit depth (1cm) of excess rainfall (runoff) occuring uniformly over the basin at a uniform rate for a specified duration.
- UH ⟶ Runoff = 1cm
  DRH ⟶ Runoff = R cm —— runoff depth ⟶ { can find from ϕ index method
  $R = P - \phi t$ (or) by volume }
- Ordinate of UH = $\frac{\text{ordinate of DRH}}{R \text{ cm}} \times 1$ cm
- Peak of UH = $\frac{\text{Peak of DRH}}{R \text{ cm}} \times 1$cm
- Ordinate of DRH = $\frac{\text{ordinate of UH}}{1 \text{cm}} \times R$ cm
- Peak of DRH = $\frac{\text{Peak of UH}}{1\text{cm}} \times R$cm
- Area of DRH = Vol. of runoff due to R cm runoff depth
  Area of UH = " " " " " 1 cm " "
- Area of DRH = (Catchment Area) × (Runoff Depth) → R cm (convert to m)
  Area of UH = (" ") × (0.01m)



- For duration D hours, average intensity of rainfall is 1/D cm/hr.

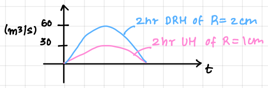
Assumptions of unit hydrograph

- Time Invariance



For same catchment and same duration ⇒ same UH
It will not vary with time

- Linear Response



$y_{DRH} = y_{UH} \times R$ ↓ runoff depth

r_1 runoff depth ⟶ y_1 ordinate
r_2 runoff depth ⟶ y_2 ordinate
(r_1+r_2) runoff depth ⟶ (y_1+y_2) ordinate

- Effective rainfall is uniformly distributed over the entire catchment area
- Rainfall Intensity is constant during the storm period.
- Unit hydrograph cannot give reliable result for basin > 5000km² or < 2 km².

Flood Hydrograph ⟶ Unit Hydrograph

1. Seperate baseflow from a flood hydrograph to get DRH
   ordinate of DRH = ordinate of FH − Baseflow
2. Calculate Runoff volume = Area of DRH = (Σ ordinates) $\times \Delta t$
3. Runoff Depth, R = $\frac{\text{Runoff Volume}}{\text{Catchment Area}}$
   ↪ rainfall excess
4. Ordinate of UH = $\frac{\text{Ordinate of DRH}}{R \text{ cm}} \times 1$ cm ; Peak of UH = $\frac{\text{Peak of DRH}}{R \text{ cm}} \times 1$cm

Unit Hydrograph ⟶ Flood Hydrograph

Data given: Runoff depth (Rcm), Baseflow, Ordinates of UH

1. Ordinates of DRH = $\frac{\text{Ordinates of UH}}{1\text{cm}} \times$ Rcm
2. ordinates of FH = ordinates of DRH + Baseflow

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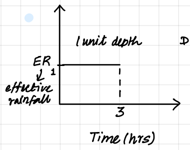
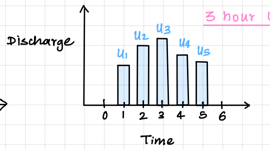




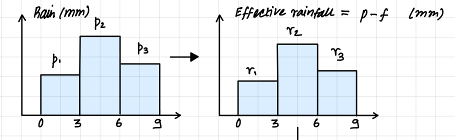
3 hour UH

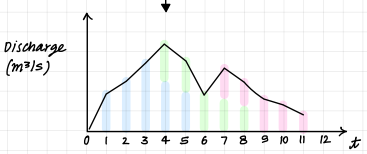
UH → DRH

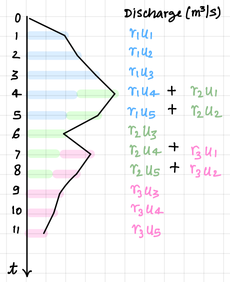
imp.

Unit Hydrograph (UH)
↓
1 Q in midsem
just now made QP

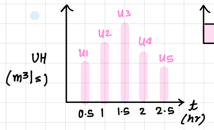
VT Chow
Applied Hydrology
Ch 1-5, 7 "must do"
★ – solved examples
Evaporation + UH







- Total volume = Area × (1 mm) — runoff depth, convert to m
- 1 mm effective rainfall (runoff depth) for unit hydrograph.
- defined for 1 mm depth, not rate.
- For hourly rate we need Greenampt eqn. Can't use CN method since it relates eff. and total rainfall in cumulative (total).
- It assumes linearity. at t = 0, discharge = 0



$$\underbrace{(u_1+u_2+u_3+u_4+u_5)}_{\text{Discharge }(m^3/s)} \times \underbrace{0.5\times60\times60}_{\text{time interval }(s)} = \underbrace{\overset{\text{catchment Area}}{\text{Area}} \times (1/1000)}_{\text{Volume }(m^2\times m)}$$

Unit Hydrograph of Different Duration

D hour UH ⟶ nD hour UH

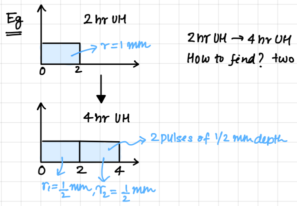
1. Method of superposition — $n = 1, 2, 3, \ldots n$ (an integer)
2. S-curve method — $n \neq 1, 2, 3, \ldots n$ (rational no. but not an integer) $n = 1/2, 1/3, 5/4, 1/4, 3/2$, etc

Method of Superposition

1. D hr UH ⟶ 2D hr UH
2. Draw UH of D hr and another D hour UH by lagging D hr.
3. Add ordinates of both to get 2 unit of 2D hr DRH.
4. Divide ordinate by 2 ⟶ 2D hr UH.

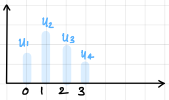
Note: As duration of UH ↑es, its peak ordinate ↓es and base period ↑es.

Eg

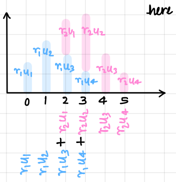


2hr UH → 4 hr UH

How to find? two storms of 1/2 mm runoff depth



here $r_1 = r_2 = 1/2$ mm



- $\text{Runoff Rate} = \dfrac{(\text{Runoff depth})}{4 \text{ hrs}}$
- Runoff depth is there, not rate. multiply by that depth not rate

- Can we make 3hr UH from 2hr UH? No, we can't use this superposition method for converting that.
  ↓
  We need to use S-curve method for that.

- In UH, we need to only care for total depth to be 1 mm.
- rainfall rate — infiltration rate
- Assume catchment doesn't change with time.

Example Rainfall of magnitude 38 cm and 28 cm occurring in two consecutive 4 hr durations on CA = 27 Km² produced following hydrograph of flow at outlet of catchment if baseflow = 5 m³/s. Find Rainfall excess.

Solⁿ

	FH		DRH
Time (hr)	Discharge (m3/s)	Baseflow (m3/s)	ordinates
0	5	5	0
6	13	5	8
12	26	5	21
18	21	5	16
24	16	5	11
30	12	5	7
36	9	5	4
42	7	5	2
48	5	5	0

→ DRH = FH − Baseflow

Area of DRH = Vol. Runoff = $(\Sigma \text{ordinates})\Delta t = (0+8+21+16+7+4+2+0)\times 6\times60\times60$
$= 1.4904 \times 10^6\ m^3$

$\text{Runoff Depth} = \dfrac{\text{Runoff Volume}}{CA} = \dfrac{1.4904\times10^6\ m^3}{27\times10^6\ m^2} = 0.0552\ m = 5.52\ cm$

$\phi \text{ index} = \dfrac{P-R}{Te} = \dfrac{3.8+2.8-5.52}{4\times2} = 0.135\ cm/hr = 1.35\ mm/hr$

Example Flood data and base flow are estimated for CA = 600 Km². Estimate excess rainfall.

	FH		DRH
Time ($\overset{day}{\cancel{hr}}$)	Discharge (m3/s)	Baseflow (m3/s)	ordinates
0	20	20	0
1	63	22	41
2	151	25	126
3	133	28	105
4	90	28	62
5	63	26	37
6	44	23	21
7	29	21	8
8	20	20	0
9	20	20	0

Σ = ▨

— DRH = FH − Baseflow

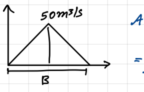
$\text{Runoff depth} = \dfrac{\overset{=\text{Area}}{\text{Runoff Vol.}}}{CA}$
$= \dfrac{(\Sigma \text{DRH}) \times 1\times60\times60\times24}{600\times10^6}$
$= 0.0576\ m$
$= 57.6\ mm$

Example The peak of FH due to 3 hr isolated storm in a catchment is 270 m³/sec. The total depth of rainfall = 59 cm. Assume average infiltration loss 0.3 cm/hr. and constant baseflow of 20 m³/s. Find peak of 3 hr UH of this catchment. If CA = 567 Km². Find base width assuming 3hr UH to be Δ.

Solⁿ :- Peak of FH = 270 m³/sec

$\text{Peak of 3hr UH} = \dfrac{\text{Peak of FH} - \text{Baseflow}}{\text{Runoff Depth}} = \dfrac{270-20}{5}\ m^3/s = 50\ m^3/s$

$R = 59 - 0.3\times3 = 5cm$



$\text{Area of UH} = C.A. \times \dfrac{1}{100} \Rightarrow B = \dfrac{567\times10^6\times10^{-3}}{25\times} = \underline{\underline{6.3\ hours}}$

$= \frac{1}{2}\times B\times 50$

Example The ordinate of 1 hr UH at 60 min interval are 0, 3, 12, 8, 6, 3 & 0 m³/s. A 2 hr storm of 4 cm excess rainfall occurred in basin from 10 AM. Consider constant baseflow 20 m³/s, the flow in river at 1 PM will be ______ ?

Solⁿ

	Time (hr)	1 hr UH	lagged 1 hr UH	2 hr DRH	2 hr UH
10AM	0	0		0	0
11	1	3	0	3	1.5
12	2	12	3	15	7.5
1 PM	3	8	12	20	10
2	4	6	8	14	7
3	5	3	6	9	4.5
4	6	0	3	3	1.5
5	7		0	0	0

$\text{DRH} = 10 \times \dfrac{\overset{\text{runoff (excess rainfall)}}{4\ cm}}{1\ cm} = 40\ m^3/s$

FH = DRH + Baseflow
= 40 − 20 = 20 m³/s

Example The ordinate of 6 hr UH are given. If two storms each 1 cm rainfall excess and 6 hr duration occur in succession. Calculate resulting FH. Assume baseflow 10 m³/s.

12 hr FH = 6hr UH + lagged 6hr UH + BF

12 hr UH → $\dfrac{FH - BF}{2}$

Time (hr)	6 hr UH	lagged 6 hr UH	Baseflow	12 hr FH	12 hr UH
0	0		10	10	0
6	20	0	10	30	10
12	60	20	10	90	40
18	150	60	10	220	105
24	120	150	10	280	135
30	90	120	10	220	105
36	66	90	10	166	78
42	50	66	10	126	58
48	32	50	10	92	41
54	20	32	10	62	26
60	10	20	10	40	15
66	0	10	10	20	5
72		0	10	10	0

Example Using the 2hr UH given below, find peak flow resulting from successive 2 hour period of rainfall 0.4, 0.9, 1.2 cm of runoff from a basin. No baseflow

Time (hr)	$\overset{2}{\cancel{1}}$ hr UH	ordinate due to 0.4 cm runoff	ordinate due to 0.9 cm runoff	ordinate due to 1.2 cm runoff	Discharge
0	0	0			0
1	58	23.2			23.2
2	173	69.2	0		69.2
3	337	134.8	52.2		187
4	440	176	155.7	0	331.7
5	285	114	303.3	69.6	486.9
6	215	86	396	207.6	689.6
7	165	66	256.5	404.4	726.9
8	90	36	193.5	528	757.5
9	35	14	148.5	342	504.5
10	0	0	81	258	339
11			31.5	198	229.5
12			0	108	108
13				42	42
14				0	0

Peak flow = 757.5 m³/s

Example Find peak of DRH due to 3 successive 2hr rainfall if runoff intensity is 0.8 cm/hr, 1 cm/hr, 1.2 cm/hr resp.

Solⁿ Runoff depth
= 0.8 cm/hr × 2 hr = 1.6 cm
= 1 " × 2 " = 2 cm
= 1.2 " × 2 " = 2.4 cm

Time (hr)	2 hr UH	ordinate due to 1.6 cm runoff	ordinate due to 2 cm runoff	ordinate due to 2.4 cm runoff	Discharge
0	0	0			0
1	6	9.6			9.6
2	10	16	0		16
3	12	19.2	12		31.2
4	18	28.8	20	0	48.8
5	10	16	24	14.4	54.4
6	6	9.6	36	24	69.6
7	4	6.4	20	28.8	55.2
8	0	0	12	43.2	55.2
9			8	24	32
10			0	14.4	14.4
11				9.6	9.6
12				0	0

Peak DRH = 69.6 m³/s

Example 2hr UH ⟶ 6 hr UH $n = \frac{6}{2} = 3$

6 hr DRH → DRH = Σ(UH values)

6 hr UH ~ $= \frac{6 \text{hr DRH}}{3}$

Time (hr)	2 hr UH	lagged by 2 hours	lagged by 4 hours	6 hr DRH	6 hr UH
0	0			0	0.00
2	20	0		20	6.67
4	10	40	0	50	16.67
6	12	20	48	80	26.67
8	18	24	24	66	22.00
10	10	36	28.8	74.8	24.93
12	6	20	43.2	69.2	23.07
14	4	12	24	40	13.33
16	0	8	14.4	22.4	7.47
18		0	9.6	9.6	3.20
20			0	0	0.00
22				0	0.00
24				0	0.00

Example 3 hr UH ⟶ 6 hr UH $n = \frac{6}{3} = 2$

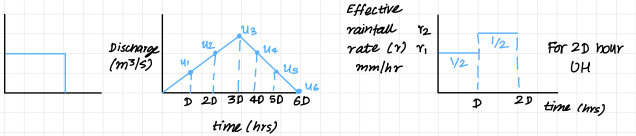
6 hr UH → $= \frac{DRH}{2}$

Time (hr)	3 hr UH	lagged by 3 hours	6 hr DRH	6 hr UH
0	0		0	0
3	10	0	10	5
6	25	10	35	17.5
9	20	25	45	22.5
12	16	20	36	18
15	12	16	28	14
18	9	12	21	10.5
21	7	9	16	8
24	5	7	12	6
27	3	5	8	4
30	0	3	3	1.5
33		0	0	0

26 Sept

Runoff

Unit Hydrograph



$$\boxed{Q_t = \sum_{m=1}^{t} E R_m U_{t-m+1}}$$

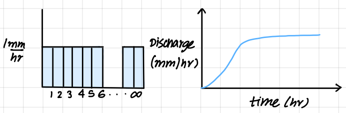
Ques.

Time	Contribution from DRH		
0	r_1	r_2	
D	$r_1 D U_1$	0	$r_1 D U_1$
2D	$r_1 D U_2$	$r_2 D U_1$	$r_1 D U_2 + r_2 D U_1$
3D	$r_1 D U_3$	$r_2 D U_2$	$r_1 D U_3 + r_2 D U_2$
4D	$r_1 D U_4$	$r_2 D U_3$	
5D	$r_1 D U_5$	$r_2 D U_4$	
6D		$r_2 D U_5$	
7D			
8D			

if $r_1 = r_2 = 1/2$ (For 2D unit hydrograph)

S-Hydrograph

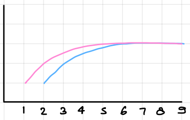
Theoretical hydrograph resulting from a continuous effective rainfall at a rate 1 mm/hr for an infinite period.



u_1	$S_1 = u_1 D$
u_2	$S_2 = u_1 D + u_2 D$
u_3	$S_3 = u_1 D + u_2 D + u_3 D$
u_4	$S_4 = u_1 D + u_2 D + u_3 D + u_4 D$
u_5	$S_5 = u_1 D + u_2 D + u_3 D + u_4 D + u_5 D$
u_6	$S_6 = u_1 D + u_2 D + u_3 D + u_4 D + u_5 D + u_6 D$

After this value of S-curve remains constant

How to find D unit hydrograph from S unit hydrograph?



D unit hydrograph lags S-hydrograph by D-time

$$U_T(t) = \frac{S(t) - S(t-T)}{T}$$

Example 2 hour UH ⟶ 3 hour UH

plot

Time	Discharge U	Solⁿ: S-hydrograph	S (m³/s)
0	0	0	0
2	2	2×2 = 4	4
4	5	4 + 5×2 = 14	14
6	7	14 + 7×2 = 28	28
8	6·5	28 + 6·5×2 = 41	41
10	3·5	41 + 3·5×2 = 48	48
12			48
14			48

Time (hr)	2 hr UH	S(t)	S(t-3) lag by 3 hrs	3 hr UH = [S(t) - S(t-3)] / 3
0	0	0	0	0.00
1		2	0	0.67
2	2	4	0	1.33
3		9	0	3.00
4	5	14	2	4.00
5		21	4	5.67
6	7	28	9	6.33
7		34.5	14	6.83
8	6.5	41	21	6.67
9		44.5	28	5.50
10	3.5	48	34.5	4.50
11		48	41	2.33
12		48	44.5	1.17
13		48	48	0.00
14		48	48	0.00

subtract and divide by 3

In S(t) calculate discharge at unknown times (1, 3, 5, etc...) by taking average of prev. & next values in general using interpolation tecniques

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CE361A

ENGINEERING HYDROLOGY

Prof. Saumyen Guha

Aman

Runoff, Stream Flow, Flood and Data Analysis

Prof. S. Guha
Class – 1

Catchment
Basin
Watershed

Flow – function of space and time
Flow at a point – time

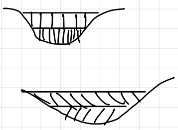
Flow ─ high rain low duration
└ low rain high duration

Try to work with finest resolution possible.

To design bridge – need max. water ~ * Annual Maximum
* min. 30 years. (long records)

$Q = \int v \, dA$

$\Sigma \bar{v}_i \, dA_i$
└ average velocity – How to measure

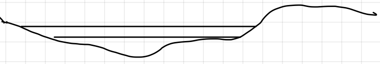


$$\bar{v} = \frac{1}{d}\int_0^d v\,dz = \frac{v_{0.2d} + v_{0.8d}}{2}$$

For shallow channels, lower
if total depth < 2·5 m

$Q = f(h)$

River Flood Plain



Indian Water Year June to May.

Flood Analysis

1. Flow dependability : Flow Duration Curve
2. Reservoir capacity : Flow Mass Curve
3. Flood estimation : Flood frequency analysis.

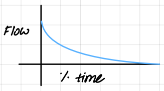
Types of Flow Analysis

3 Oct

1. Flow Duration Curve
2. Flow Mass Curve
3. Flood Frequency Analysis — 2 to 3 classes

- PDF, RV, etc — brush up.

Flow Duration Curve



- Provides an idea about
- No chronology
- No data information
- Itself an incomplete graph

- can be prepared for any time series

- hydroelectric power — max. annual discharge needed for it.
  - * firm power ~ 95% Dependability (Flow duration curve) (primary)
  - * secondary power b/w upper & firm power

η = unit wt. of
ξ = eff. of hydrolic power plant
H = head diff.

Eqⁿ ① slides

Formula

$$\xi = \frac{\eta \gamma H}{1000}(QH - Q_{95}H_{95})$$

Steps to make Flow Duration Curve

1. Arrange the data in ↓sing order of magnitude
2. $p = \frac{m}{N+1} \times 100$

N = total no. of data
m = hierarchy position (rank)

What happens when two values are same? either same rank or diff. rank.
Unrealistic - just a monthly curve

Flow Mass Curve

	Q_v (mm³)	cumulative vol. flow (ΣQ_v)
Jun ___	x_1	x_1
July ___	x_2	$x_1 + x_2$
Aug ___	x_3	$x_1 + x_2 + x_3$
___	x_4	
___		
July ___		

- Similar to Precipitation mass curve

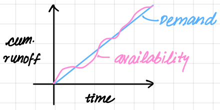
- $CR = \int_0^t Q\,dt$

Availability curve

ha m = 10^4 m³
ha = 10^3 m

- $CD = \int_0^t D\,dt$

Demand curve



- When superimpose demand curve to availability curve? above and lower.

- Non-linear mostly (never linear mostly)

$$\left.\begin{aligned}\frac{d}{dt}(CR) &= Q\\ \frac{d}{dt}(CD) &= D\end{aligned}\right\}$$ slopes of curves

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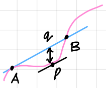


A → Demand = Availability
Right of A → Demand > Runoff → Reservoir should supply water
p → Q = D again
min. reservoir capacity is dist. b/w q and p.

Variable Demand

Month	QV	DV	QV − DV	cum. def.	cum. excess
—	—	—	−ve	−ve	|
—	—	—	+ve	|	+ve
—	—	—	+ve	|	+ve
—	—	—	+ve	|	+ve
—	—	—	−ve	−ve	|
—	—	—	−ve	−ve	|
—	—	—	−ve	−ve	|

segment calculation method

- Minimum Reservoir Capacity (supply deficit) = max (cum. def.) = 335 for this data

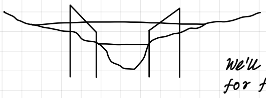
- For Reservoir Planning — need at least 2 years data — more the better.

Imp. Sequent Peak Algorithm comes in every competitive examination

Assume 2 year period repeats.
Fourier Period repeats is the assumption here. (Fourier Series)

Capacity = max. b/w peak (P_i) and trough (T_i) is the reservoir capacity.

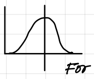
Flood Frequency Analysis



Channel flow

We'll be talking about channel flow (max.) for flood reference.

Concerns
1. Peak Discharge
2. Time of arrival of peak discharge
3. Stage of the peak discharge



For flood analysis: —
1. Given Return Period → Discharge
2. Given Discharge → Return Period

Outline of Statistical Analysis

X : magnitude of flood / flow → Random Variable
X = x
(x_T) $P(X \geq x) = p$ (exceedence probability)

Derivable

* $\boxed{\langle T \rangle = \frac{1}{p}}$

Lesser time averaging, higher the peak
Annual maximum

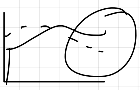
↑ daily
10 d
15d
monthly
annually

- finer the resolution → more the annual max. discharge value
- Ven Te Chow — Partial Duration Series. / Annual exceedence series.
- Extreme value means max. or min.
- Preprocess the data & prepare time series
- Selection of method — non-parametric — used in 1890s / parametric — done in today's world
- maximum likelihood — brush up your basic statistics
- Goodness of fit. — Graphical Comparison / χ^2 test

K.S. test (easiest) — but we won't do that

Example
1. Sheet Data
2. Time Series Data ——— mean ↑↓ std. dev. ↓↑
3. Histogram - Freq. vs time ——— symmetric & non-symetric

How to compute P? (Non-parametric method)
1. Arrange in ↓sing order
2. rank, 1-largest, N-smallest
3. $p = \frac{m-b}{N+1-2b}$ $b \in (0,1)$ depending on method. — 0 weibul / 0.5 hazen / 1 modal
4. If few equal assign highest rank to all of them.
5. $p = 1/T$



can predict any year — parametric
can't predict all years — non-parametric

Parametric Method — Real thing ~ Fitting probability distribution

- Normal $X \sim N(\mu, \sigma)$
- Log Normal $Y = \ln X$ is log normally
- Gumbels
- Log Pearson Type III / Gamma distribution.

Parametric Methods Probability Distributions

How to fit what distribution to data? Introduce the distributions
continuous vs discrete

X. x.
$P(X = x)$
✓ discrete ↓ can find
× continuous ↓ = 0 (can't find)

$P_A = \int_A f(x)dx$

A C Domain

$P(X < x) = P(X \leq x)$
$(x_1 < x < x_2)$
$(x_1 \leq x < x_2)$
$(x_1 < x \leq x_2)$
$(x_1 \leq x \leq x_2)$

- Normal Distribution
- Central Limit Theorem
  - most essential theorem

iid (independent identical distribution)
sum them all, you get a normal distribution.

→ Log-Normal Distribution (2 parameters)
→ Extreme Val. Dist. Type I or Gumbel Distribution (2 parameter)
→ Log Pearson Type III Dist. (3 parameters)

cts & uniformly cts.
difference b/w them.

$$E[g(x)] = \int_{-\alpha}^{\alpha} g(x)f(x)dx$$

$\frac{\sigma}{\sqrt{n}}$

- Varience
- Std. Deviation
- Skewness ~ 3rd moment
- Kurtosis ~ 4th moment

feynman - difference b/w knowing and understanding
Bird names only but no habbits
Bird habits and no name

μ = — lost one degree of freedom

Devore ≃ Sir's Favourite UG level.

$s = \frac{=}{N-1}$ lost one more degree of freedom / information

$y = \frac{n\Sigma ==}{(N-1)(N-2)}$ lost one more degree of freedom / information.

$$\left.\begin{aligned}\alpha &= f(\mu, \sigma)\\ \beta &= g(\mu, \sigma)\end{aligned}\right\}$$

- No free lunch!

Normal Distribution

PDF
CDF
Mean Median Mode
Skewness
Std. dev.

error function
└ in excel also there.

erf.
erfc.

error funcⁿ

matlab, excel
must know these two " " program.

μ_y, σ_y^2 independently affects normally distributed.

$$\sigma_x = \exp\left(\mu_y + \frac{\sigma_y^2}{2}\right)\left(\sqrt{\exp(\sigma_y^2)} - 1\right.$$

$$\int_{-\alpha}^{\alpha} x f(x) dx$$

$\bigcirc{e^2}$ intersecting $y = 2$ mean, $\underline{\underline{x = e^2}}$.

---

x Year $\quad$ X $\quad$ $Y = \ln X$

× $\boxed{\mu_x \; \sigma_x \; \gamma_x}$ ✓ $\bar{x}$ S_x Sk_x ⤳ can estimate these

× $\boxed{\mu_y \; \sigma_y \; \gamma_y}$ ✓ $\bar{y}$ S_y Sk_y

↳ Never compute this (fundamental concept)

Either assume this or correct for finite data size.

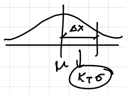
$$E[g(x)] = \sum_{i=0}^{\infty} g(x) f(x)$$

↳ $(1-p)^{T-1} p$.

$x \sim F(x)$
↑

K_T → Frequency factor
Confidence Interval

T
$X = x_T$



$$K_T = \frac{x_T - \mu_x}{\sigma_x}$$

K_T = +ve or −ve value — sir don't use (±)

Derive the expression for K_T (hw)

Normal gives −ve flood values so normally not used for flood

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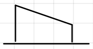
9 Nov

Darcy's law
$(Q = KiA)$
i = hydraulic gradient $= \frac{\Delta h}{L} = \frac{h_1 - h_2}{L}$

det.



q = darcy flux ≠ seepage velocity → $= \left(\frac{q}{n_0}\right)$

darcy is actually momentum conservation.
↳ can be derived from navier strokes.

mass
momentum
energy

K = hydraulic conductivity [LT]
Dimension

- Yes, Darcy's Law is momentum conservation
- K = hydraulic conductivity
  K = same for same solid & fluid properties.

For same solid/fluid, K is same.

- $k = L^2$ $\quad$ k = intrinsic permeability
- S = dimensionless.
- Transmissivity is only property of unconfined aquifer.