CE481A

TRANSPORTATION FACILITIES DESIGN

PART-2 TRAFFIC DESIGN

Dr. Venkatesan Kanagaraj

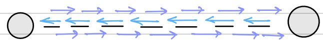
Quiz-1 Assignment – Not graded
End sem

Self organized Behavior of Army Ants in bidirectional traffic

'Pheromone'

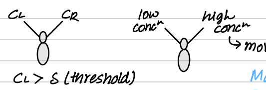
Army Ants

- Army ants are almost completely blind.
- They interact through 'Pheromone' – a chemical substance.
- They loosely form 3 lanes
- Pheromone defuse and decay.
- Army ants move from low concⁿ to high concⁿ.
- Inbound ants release high concⁿ as compared to outbound ants.
- Pheromone evaporates as well.



Inbound ant
Outbound ant

pheromone
Inbound – high concⁿ
outbound – low concⁿ
$c_I > c_O$



↳ move here

$C_L > \delta$ (threshold)
then move there

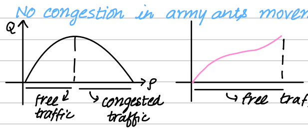
Modeling of ants traffic

- Model for Pheromone
- Traffic Flow for army ants

Foraging in army ants

- Scout ants explore the environment to find food.
- Fixed scouts for colony sizes.
- Interaction through pheromone only.

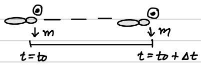
No congestion in army ants movement



army ants
→ No congestion happens
→ congested graph always absent

Assumptions

- Pheromone — point source
  (finite amount of pheromone)



$m_I > m_O$ (inbound > outbound)
For N ants, N×m mass concⁿ of pheromone at each time stamp

- Pheromone – defuse and decay – PDE equation

$$\frac{\partial c}{\partial t} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right)$$ (diffusion model)

D → diffusion coeff. — model parameter

Boundary Conditions:- $x: -\infty$ to $+\infty$, $y: -\infty$ to $+\infty$ → Analytical solⁿ — linear PDE (lucky) / Numerical solⁿ

$-\infty$ to $+\infty$ → Apply 2 dimensional fourier transform.

Decay:- $c_i \times e^{-\gamma(t-t_0)}$ (γ → model parameter)

Mathematical model for ant movement
Self driven particle model
↳ similar to car following model
self propelling force

$$\frac{dv_i}{dt} = \vec{F_d} + \sum_{j=1}^{n} \vec{F_{ij}}^{ph} + n_i$$

$\vec{F_d}$ → self propelling force (pheromone concⁿ); $\vec{F_{ij}}^{ph}$ → physical force → δ↓ physical force, δ↑ force; n_i → noise/error term

v_x v_y – both dirⁿ

$$\vec{F_D} = \frac{v_i^0 \vec{d_i} - \vec{v_i}}{\tau}$$ → car following model formula
common for many others

v_i^0 → depends on pheromone concⁿ
$\vec{d_i}$ → desired directional vector

Have to find? — desired directional vector ($\vec{d_i}$) — desired speed (v_0)

$c < \varepsilon$ — very small concentration — no detection

$c > \delta$ — saturation (high concentration)
After saturation, doesn't matter how much you release pheromone, no impact on ant speed due to no detection.

eg:- $\delta = 10$ — 20, 40 — no matter same speed of ant

Use of $$v_i^0(c_p) = v_i^{max}\frac{\left(\tanh\left(\frac{c_p}{\Delta c_p} - \beta\right) + \tanh\beta\right)}{1+\tanh\beta}$$

———×———×———×———×———

This course →

Fundamental Diagram

$$\left[\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0\right]$$ continuity equation
↳ Non-linear

1. Numerical Methods ~ Not in syllabus but real learning in engineering.
   - Power Series
   - Taylor Series — remainder term
   - Finite Difference Method - can't solve this – why? – derivative don't exist
   - Finite Volume Method
   - Good and over scheme
     → Self transmission model — common model in traffic flow theory.
2. Traffic Signs, Signals, etc
   Apply formula and questions — toll plazas, etc . . .

CE481A

30 Sept

Power series

1. convergence
2. divergence

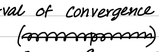
$$\sum_0^\infty c_n(x-a)^n \longrightarrow \text{Infinite series}$$

a → center of P.S.

$n = 0, 1, 2, 3, \dots$

$c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$

1) Radius of convergence
   $|x-a| < R$
2) Interval of Convergence



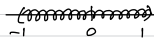
tests for convergence?
How to know its convergence

$1 + x + x^2 + \dots$ → center of P.S. $= 0$, $c_n = 1$

$$\Big\} = \frac{1}{1-x}, \quad |x| < 1$$ (infinite GP $\Sigma = a/(1-r)$)

$$\sum_0^\infty c_n(x-a)^n$$

$a = 0$



↳ not including end points.

$|x-a| < R$
$|x-0| < 1$
$|x| < 1$

Mathematically how to ensure its convergence

Partial sums
$S_0 = c_0(x-a)^0 = c_0$
$S_1 = c_0 + c_1(x-a)$
$S_2 = c_0 + c_1(x-a) + c_2(x-a)^2$
⋮

truncated power series

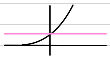
$S_n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots + c_n(x-a)^n$

How to define its convergence?

$$\boxed{\lim_{n\to\infty} S_n = S}$$ → some finite value

then we say that this power series will converge.

Taylor series



$e^0 = 1$ $\quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$e^{0.2} = ?$

$e^{0.2} \approx 1$ matching same value at $x=0$

Taylor series $e^x = 1$

Taylor series $e^x = 1 + x$

$e^x \approx c_0 x^0$ $\quad a = 0$ $\quad \sum_0^\infty c_n (x-a)^n$ $\quad e^x = \sum c_n (x-a)^n = \sum_0^\infty c_n x^n$

$e^0 = c_0$ $\quad e^x \approx 1$ $\quad$ linear approximation

$c_0 = 1$

$e^x = c_0 + c_1 x$

$\frac{d}{dx} e^x = 0 + c_1$

$e^0 = 1 = c_1$

$e^x \approx 1 + x$

$e^x = c_0 + c_1 x + c_2 x^2$

$\frac{d}{dx} e^x = c_1 + 2c_2 x$ $\quad \rightsquigarrow e^x = 1 + x + \frac{x^2}{2!} \cdots$ and so on

$\frac{d}{dx} e^x = 2c_2$ $\quad c_2 = 1/2$ $\quad \rightarrow x \in R$ $\quad (-\infty, \infty)$

$\rightarrow$ Radius of convergence is ∞.

<u>For general functions</u>,

$f(x) = \sum_0^\infty c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$

$f \rightarrow$ continuous

$f \rightarrow$ infinitely many times differentiable

$\underline{x = a}$

$f(a) = c_0 + 0 + + 0 + \cdots$ $\quad c_0 = f(a)$

$f'(x) = c_1 + 2c_2(x-a) + \cdots$

$f'(a) = c_1 + 0 + 0$ $\quad c_1 = f'(a)$

$f''(x) = 2c_2 + 3 \times 2 c_3 (x-a) + \cdots$

$f''(a) = 2c_2$ $\quad c_2 = \frac{f''(a)}{2}$ $\quad c_n = \frac{f^n(a)}{n!}$

$$\boxed{f(x) = \sum_{n=0}^\infty \frac{f^n(a)}{n!} (x-a)^n}$$

$c_n = \frac{f^n(a)}{n!}$ $\quad |x-a| < R$

$a-R \quad a \quad a+R$

Maclaurin series

If $a=0$ then taylor series is called Maclaurin series.

$f(a) = \sum_{n=0}^\infty \frac{f^n(0) x^n}{n!}$ $\quad |x| < R$

$(-R, R)$

$c_n = \frac{f^n(a)}{n!}$

Taylor's Theorem

$f(x) = \sum \frac{f^n(a)}{n!} (x-a)^n$ $\quad$ Infinite series

Taylor Polynomial — truncation of taylor series

$P_0 = f(a)$

$P_1 = f(a) + f'(a)(x-a)$

$P_2 = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2$

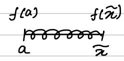
$\vdots$

$P_n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$ $\quad c_n \neq 0$

n^{th} order taylor polynomial

$f(x) = P_n + R$ $\quad \hookrightarrow$ Remainder term (Error term)

$\downarrow$ upto some finite order



$f(x) = P_0 + R$

$f(x) = f(a) + R$

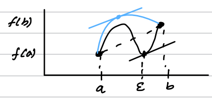
$f(a) = f(a) + 0 \hookrightarrow$ zero remainder term $\quad$ have to match at a & $\tilde{x}$

$f(\tilde{x}) = f(a) + R$ $\quad$ not neccesarily b/w them.

Lagrangian Mean Value Theorem (MVT)

$f \rightarrow$ continuous $\quad f \rightarrow$ differentiable

$f'(\mathcal{E}) = \frac{\Delta f}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ $\quad \mathcal{E} \in (a,b)$



$f(b) = f(a) + f'(\mathcal{E})(b-a)$

$f(x) = f(a) + R$

$f(x) \approx f(a) + c(x-a)$ $\quad \leftrightarrow$ construct c $\tilde{c}$

$f(\tilde{x}) = f(a) + \tilde{c}(x-a)$

$D(x) = f(\tilde{x}) - f(a) - c|\tilde{x} - a|$ $\quad$ (Difference b/w original and approx. function)

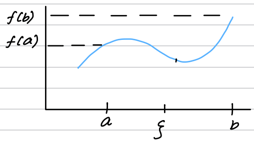
$\left.\begin{array}{lll} f(a) & D(x)=0 & x=a \\ f(\tilde{x}) & D(x)=0 & x=\tilde{x} \end{array}\right\}$

For $D(x) = 0$, $\quad c = \frac{f(x)-f(a)}{x-a} = f'(\xi)$ (using MVT)

$c = f'(\xi)$

5 Oct

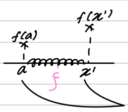
Mean Value Theorem



$\frac{\Delta f}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ $\quad \xi \in (a,b)$

$f'(\xi) =$ average slope $\frac{\Delta f}{\Delta x} = \frac{f(b)-f(a)}{b-a}$

Taylor Series:



exactly match at these pts.
other points, not necesary to match.

$f(x) \approx f(a) + \underbrace{\frac{c}{1!}(x-a)}_{\text{Remainder term}}$

$D(x) = f(x) - f(a) - \frac{c}{1!}(x-a)$

$x = a$ $\quad 0 = f(a) - f(a) - 0$

$0 = f(x') - f(a) - c(x'-a) \Rightarrow c = \frac{f(x')-f(a)}{x'-a} = f'(\xi)$

$f(x) \approx f(a) + \underbrace{\frac{f'(\xi)}{1!}(x-a)}$

Remainder term for 0^{th} order polynomial.

2^{nd} order

$f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{c(x-a)^2}{2!}$

$f'(x) = f'(a) + \frac{2c}{2!}(x-a)$

$D(x) = f(x) - \left[f(a) + \frac{f'(a)}{1!}(x-a) + \frac{c(x-a)^2}{2!}\right]$

$D'(x) = f'(x) - f'(a) - c(x-a)$

$c = \frac{f'(x') - f'(a)}{x'-a} = f''(\xi)$

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \underbrace{\frac{f''(\xi)}{2!}(x-a)^2}_{\text{Remainder Term}}$

For N^{th} order polynomial

$f(x) = P_n(x) + R_n$

$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$

$+ \frac{f^{n+1}(a)}{(n+1)!}(x-a)^{n+1}$

at least $n+1$ differentiable.

<u>Example</u>: e^x $\quad a = 0$

$e^x = \sum_{i=0}^\infty \frac{x^i}{i+1}$ $\quad x \in R$ $\quad \frac{d^n}{dx^n} e^x = e^x$

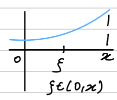
$e^x = P_n(x) + R_n$

$e^x = \sum_{i=0}^n \frac{x_i}{i!} + R_n$ $\quad \{\because f(a) = 1 = f'(a) = f''(a) = \cdots f^n(a)\}$

$n \to \infty$, $R_n \to 0$

series converges

$R_{n+1} = \frac{f^{n+1}(\xi)}{(n+1)!}(x-a)^{n+1} = \frac{e^\xi}{(n+1)!}(x-a)^{n+1}$



$\lim_{n\to\infty} R_{n+1} = 0$

$\lim_{n\to\infty} \frac{e^\xi (x-a)^{n+1}}{(n+1)!} = e^\xi \lim_{n\to\infty} \frac{x^{n+1}}{(n+1)!} \longrightarrow 0$

converges on entire real line.

$\frac{x^{n+1}}{(n+1)!}$ $\quad \frac{7^n}{n!}$

$\frac{7^n}{n!} = \frac{7}{n} \underbrace{\cdots}_{<1} \underbrace{\frac{7}{7}\frac{7}{6}\frac{7}{5}\frac{7}{4}\frac{7}{3}\frac{7}{2}\frac{7}{1}}_{>1}$ $\quad \lim_{n\to\infty} 0 < \lim_{n\to\infty} \frac{7^n}{n!} < \lim_{n\to\infty} \frac{7^6}{6!}\frac{1}{n}$

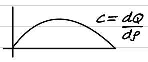
$\therefore$ Remainder term $\to 0$

Numerical Tecniques

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0$$ (Continuity Equation) — PDE

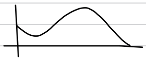
Non-Linear PDE

$$\frac{\partial \rho}{\partial t} + \frac{dQ}{d\rho}\frac{\partial \rho}{\partial x} = 0$$



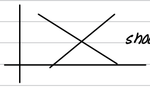
$c = \frac{dQ}{d\rho}$ $\quad Q = \rho V_f \left(1 - \frac{\rho^2}{\rho_{max}}\right)$

Why Numerical Methods?



Gradient catastrophe...

1. Non-linear PDE ⇒ shock waves and expansion waves form. that's why Num. Methods

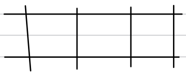


shock waves

2. Roads have different geometry that's why Num. Methods

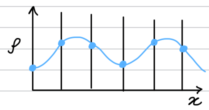
1. Finite Difference Method
2. Finite Volume Method

road is continuous



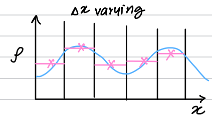
Procedure
1. Domain Discretization
2. PDE Discretization
   convert PDE to algebraic/differential equation
3. Apply Numerical Methods to solve algebraic equations
4. Feed into computer
5. Results and Model parameters —— change parameter to get satisfactory results.

Finite Difference Method (FD)



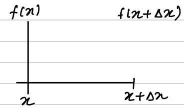
No distinct derivatives
computer only has there point
grid point = 6
continuous
function value at a grid point = known

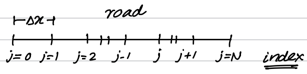
Δx varying



Average value of function in a particular grid.
This is cont. function ⟶ F.D.

Assume Δx equally divided





$$f(x+\Delta x) = f(x) + \frac{f'(x)}{1!}\Delta x + \frac{f''(x)(\Delta x)^2}{2!} + \frac{f'''(x)(\Delta x)^3}{3!}$$

$x = j \to$ space

$x + \Delta x = j+1$

$t = n \to$ time

$$f_{j+1}^n = f_j^n + \frac{\Delta x}{1!} f'(x)_j^n + \frac{(\Delta x)^2}{2!} f''(x)_j^n + \frac{(\Delta x)^3}{3!} f'''(x)_\xi^n$$

For Remainder term, ξ

$$f_{j+1}^n - f_j^n = \Delta x\, f(x)\Big|_j^n + \frac{\Delta x^2}{2} f''(x)\Big|_\xi^n$$

$$\boxed{f'(x)\Big|_j^n = \frac{f_{i+1}^n - f_i^n}{\Delta x} - \underbrace{\frac{\Delta x}{2!} f''(\xi, n)}_{\text{truncation error}}}$$

Forward Difference in space
First order approximation

→ Forward approximation in space

Truncation error = $O(\Delta x)$
order of approx^m = Δx

can't find ξ
$\Delta x \to 0$, Error $\to 0$ } order of Δx error
Big who ⟶ C.S. peeps

$$f_{j-1}^n = f_j^n - \frac{\Delta x}{1!} f'(x)_j^n + \frac{(\Delta x)^2}{2!} f''(\xi, n)$$

$$\Delta x\, f'(x)_j^n = f_j^n - f_{j-1}^n + \frac{(\Delta x)^2}{2!} f''(\xi, n)$$

Truncation Error → PDE → Algebraic Equation

$$\boxed{f'(x)_j^n = \frac{f_j^n - f_{j-1}^n}{\Delta x} + \frac{\Delta x}{2!} f''(\xi, n)}$$

≠ Round-off error (different concept)

→ Backward approximation in space

$$f_{i+1}^n - f_{i-1}^n = 2\Delta x\, f'(x)\Big|_j^n + \frac{2(\Delta x)^3}{3!} f'''(\xi, \eta)$$

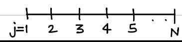
$$\boxed{f'(x)\Big|_j^n = \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x} - \frac{(\Delta x)^2}{3!} f'''(\xi, n)}$$

central Difference ⟶ Better, reduced error & fast

$$f(x+a) = f(x) + f'(x)(x-a) + \frac{f''(x)(x-a)^2}{2!} + \frac{f'''(x)(x-a)^3}{3!}$$

$$f_{j+1}^n = f_j^n + f'\Big|_j^n \Delta x + f''\Big|_j^n (\Delta x)^2 + \frac{f'''(\xi)}{3!}(\Delta x)^3$$

$$f(x-a) = f(x) - f'(x)\,\Delta x + \frac{f''(x)(\Delta x)^2}{2!} + \frac{f'''(\xi)}{3!}(\Delta x)^3$$



Analytical approximation → Truncation Errors $\boxed{\text{9 Oct}}$

F.D. $\quad \frac{\partial f}{\partial x}\Big|_i^n = \frac{f_{i+1}^n - f_i^n}{\Delta x} \quad O(\Delta x)$

B.D. $\quad \frac{df}{dx}\Big|_i^n = \frac{f_i^n - f_{i-1}^n}{\Delta x} \quad O(\Delta x)$

C.D. $\quad \frac{df}{dx}\Big|_i^n = \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x} \quad O(\Delta x^2)$

$$\frac{d^2 f}{dx^2}\Big|_i^n = \frac{f_{i+1}^n - 2f_i^n + f_{i-1}^n}{\underbrace{2\Delta x^2}_{\text{check it}}} \quad O(\Delta x^2)$$

consistency
stability
convergence

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{dQ}{d\rho}\frac{\partial \rho}{\partial x} = 0 \qquad \text{Non-Linear eq}^n$$

$\frac{dQ}{d\rho} = c = \tilde{c}$ (constant)

$$\frac{\partial \rho}{\partial t} + \tilde{c}\frac{\partial \rho}{\partial x} = 0$$

j : space index
n : time index
Forward time central space

$$\frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} + \tilde{c}\,\frac{\rho_{j+1}^n - \rho_{j-1}^n}{2\Delta x} = 0 \qquad \text{Expand using taylor series}$$

$$\frac{\rho_j^n - \rho_j^n(\Delta t) + \frac{\rho_j^{(\alpha)}}{2!}\Delta t - \rho_j^n}{\Delta t}$$

$$\rho_j'^n + \frac{\rho_j''^{(\alpha)}}{2!}\Delta t$$

$$\frac{\partial \rho}{\partial t}\Big|_j^n$$

$$\frac{\rho_j^n + \rho_j'^n(\Delta t) + \frac{\rho_j''^{(\alpha)}(\Delta t)^2}{2!\,\Delta t} - \rho_j^n}{\Delta t} \qquad \frac{\rho_j'(\Delta t)}{\Delta t} + \frac{\rho_j''^{\alpha}(\Delta t)^2}{2\Delta t}$$

modified PDE Method

$$\frac{(\rho_j^n + \rho')_j^n(\Delta t) + \rho''{}_j^n\frac{(\Delta t)^2}{2!} - \rho_j^n}{\Delta t}$$

$$\rho_j'^n + \rho''{}_j^\alpha\frac{(\Delta t)}{2!}$$

$$\frac{\cancel{\rho_j^n} + \frac{\rho'|_j^n}{1!}\Delta x + \cancel{\frac{\rho''|_j^n}{2!}}(\Delta x)^2 + \frac{\rho'''|_j^n}{3!}(\Delta x)^3 + \frac{\rho^{iv}|_\alpha^n}{4!}(\Delta x)^4 - \left(\cancel{\rho_j^n} - \frac{\rho'|_j^n}{1!}\Delta x + \cancel{\frac{\rho''|_j^n}{2!}}(\Delta x)^2 - \frac{\rho'''|_j^n}{3!}(\Delta x)^3 + \frac{\rho^{iv}|_\varepsilon^n}{4!}(\Delta x)^4\right)}{2\Delta x}$$

$$= \frac{\frac{2\rho'''}{3!}\Big|_\varepsilon^n (\Delta x)^3}{2\Delta x}$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial^2 \rho}{\partial t^2}(\Delta t) + \tilde{c}\frac{\partial \rho}{\partial x} + \frac{\partial^3 \rho}{\partial x^3}\frac{(\Delta x)^3}{3!} = 0$$

$$\frac{\partial \rho}{\partial t} + \tilde{c}\frac{\partial \rho}{\partial x} + \underbrace{\frac{\partial^2 \rho}{\partial t^2}(\Delta t)}_{(\alpha)}\,\underbrace{\tilde{c}\,\frac{\partial^3 \rho}{\partial x^3}\frac{(\Delta x)^2}{3!}}_{(\varepsilon)} = 0$$

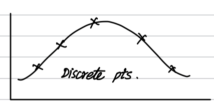
1. consistent condition

If $\Delta t \to 0$ $\quad \Delta x \to 0$ then $\alpha, \varepsilon \to 0$ $\quad$ consistency
modified PDE converts to original PDE.

2. stability → Round-off error

$\varepsilon^{n+1} > \varepsilon^n$ → Round off error

3.878745|3842 → computer truncate this number
→ Round off error



Fourier Discrete Analysis
$\Delta t, \Delta x$ condition for stability

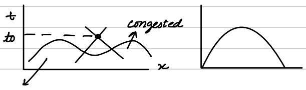
3. convergence

If consistent and stable, then modified PDE automatically converge to actual PDE. It's for linear PDE only.
For non-linear PDE it is not there, but we're assuming.

Traffic Flow part . . .

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \text{— (1)}$$

(continuity Eqⁿ)

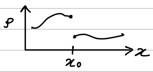


$W_s = \frac{Q_1 - Q_2}{\rho_1 - \rho_2}$ (shock wave

After shock wave

$$\frac{\partial \rho}{\partial t} + \frac{dQ}{d\rho}\frac{\partial \rho}{\partial x} \qquad \frac{\partial \rho}{\partial x} \longrightarrow \text{doesn't exist after discontinuity (shock waves)}$$

$$\frac{\partial \rho}{\partial x} = \frac{\rho_{i+1}^n - \rho_{i+1}^n}{\Delta x} + \text{Trun error}$$



can't solve using Finite Difference Method.

Non-linear hyperbolic PDE
Derivative doesn't exist
Truncation error goes to ∞

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0} \quad \text{conservative form}$$

Integrating this equation,

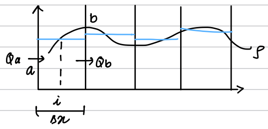
$$\int_a^b \left(\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x}\right) dx = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial \rho}\frac{\partial \rho}{\partial x} = 0} \quad \text{non-conservative form}$$

$$\frac{d}{dt}\int_a^b \rho\, dx + \int_a^b \frac{\partial Q}{\cancel{\partial x}}\, d\cancel{x}$$

$$\boxed{\frac{d}{dt}\int_a^b \rho\, dx + Q(b) - Q(a) = 0} \quad \text{Integral Form}$$

Finite Volume Method



$$\frac{d}{dt}\int_a^b \rho\, dx = \underset{\text{Flow in}}{Q(a)} - \underset{\text{Flow out}}{Q(b)}$$

$\frac{d}{dt}\int_a^b \rho\, dx$ = change of total number of vehicles

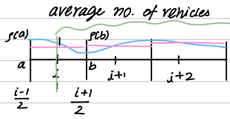
$$\frac{1}{\Delta x}\frac{d}{dt}\int_a^b \rho\, dx = \frac{Q(a) - Q(b)}{\Delta x}$$

$$\frac{d}{dt}\int \frac{\rho\, dx}{\Delta x} = \frac{Q(a) - Q(b)}{\Delta x}$$

→ Applicable to shock wave also.
→ shock wave speed — derived using integral form so F.V.M. applicable

$$\frac{d}{dt}(\tilde{\rho}) = \frac{Q(a) - Q(b)}{\Delta x}$$

average no. of vehicles → physical shock wave — it'll go cell interface.



$Q = \rho v$
$Q = f(\rho)$
$Q_1 = f(\rho(a))$
$Q_2 = f(\rho(b))$

Density → Q_1 & Q_2
only know cell average
At $t = 0$, everything given
after that find out approxᵐ

$$\frac{d\rho}{dt} = \frac{Q(a) - Q(b)}{\Delta x}$$

two approxᵐ introduced!

$$\frac{\bar{\rho}_i^{n+1} - \bar{\rho}_i^{n}}{\Delta t} = \frac{Q(a) - Q(b)}{\Delta x}$$

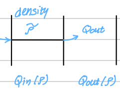
average density

12 Oct

$$\bar{\rho}_i^{n+1} = \bar{\rho}_i^{n} + \frac{\Delta t}{\Delta x}(Q(a) - Q(b))$$

$$\tilde{\rho}^{n+1} = \tilde{\rho}^{n} + \frac{\Delta t}{\Delta x}(Q_{in} - Q_{out})$$

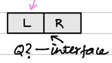
density $\tilde{\rho}^n$ at time n
$\tilde{\rho}^{n+1}$ at time n+1



$$\frac{d}{dt}\int_a^b \rho\, dx + Q(b) - Q(a) = 0 \implies \frac{1}{\Delta x}\cdot\frac{d}{dt}\int_a^b \rho\, dx = \frac{Q(a) - Q(b)}{\Delta x}$$

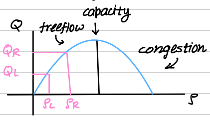
$$\frac{d}{dt}\underbrace{\left(\frac{\int_a^b \rho\, dx}{\Delta x}\right)}_{\text{average density}} = \frac{Q_{in} - Q_{out}}{\Delta x} \implies \frac{d\tilde{\rho}}{dt} = \frac{Q_{in} - Q_{out}}{\Delta x}$$

single lane road



Godunov Scheme

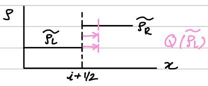
max possible $\frac{dQ}{d\rho} = c = 0$
↓
capacity

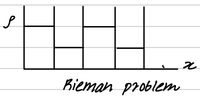


case 1

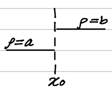
1) $\left.\frac{dQ}{d\rho}\right|_L > 0$, $\left.\frac{dQ}{d\rho}\right|_{R} > 0$

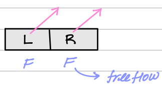
Godunov Scheme



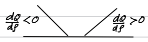


$$\rho(x) = \begin{cases} a & , x < x_0 \\ b & , x > x_0 \end{cases}$$



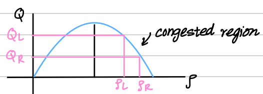


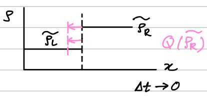
$\Delta t \to 0$, how density profile move? $c = \frac{dQ}{d\rho}$
using characteristic speed
Both characteristic speed = +ve
Both density profile move rightward.



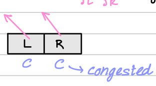
case 2

$\left.\frac{dQ}{d\rho}\right|_L < 0$, $\left.\frac{dQ}{d\rho}\right|_R < 0$



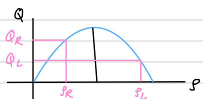


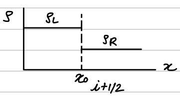
$\Delta t \to 0$

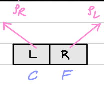


case 3 Expansion Waves

$\left.\frac{dQ}{d\rho}\right|_L < 0 < \left.\frac{dQ}{d\rho}\right|_R$

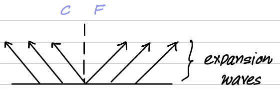






$Q_{i+1/2} = Q_{max}(\rho)$

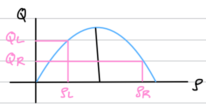
$\rho_R \leq \rho \leq \rho_L$

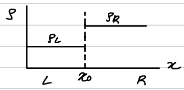


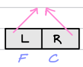
max (Q_R, Q_L) — not correct

case 4 Shock Waves

$\left.\frac{dQ}{d\rho}\right|_L > 0 > \left.\frac{dQ}{d\rho}\right|_R$







shockwave occuring

$Q_{i+1/2} = Q(\rho_R)$

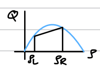
Accurate Method . . .

$$\frac{\Delta Q}{\Delta \rho} = \frac{Q_R - Q_L}{\rho_R - \rho_L} = \frac{-ve}{+ve} < 0 \quad ; c < 0$$

shockwave travelling leftward
use $Q(\rho_R)$

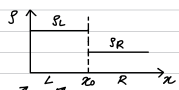
$$\frac{\Delta Q}{\Delta \rho} = \frac{Q_R - Q_L}{\rho_R - \rho_L} = +ve > 0 \quad ; c > 0$$

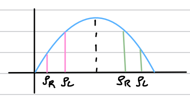
use $Q_{i+1/2} = Q(\rho_L)$



Generalise

Case 1 $\rho_L > \rho_R$
$Q = \max Q(\rho)$
$\rho_R \leq \rho \leq \rho_L$



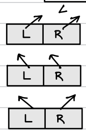


max$[Q(\rho_L), Q(\rho_R)]$

$\left.\frac{dQ}{d\rho}\right|_L > 0$ $\left.\frac{dQ}{d\rho}\right|_R > 0$

$\left.\frac{dQ}{d\rho}\right|_L < 0$ $\left.\frac{dQ}{d\rho}\right|_R < 0$

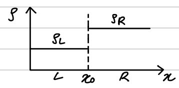
$\left.\frac{dQ}{d\rho}\right|_L < 0 < \left.\frac{dQ}{d\rho}\right|_R$



expansion waves

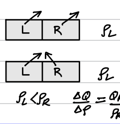
Case 2 $\rho_L < \rho_R$
$Q = \min Q(\rho)$
$\rho_R \leq \rho \leq \rho_L$

min$[Q(\rho_L), Q(\rho_R)]$



$\left.\frac{dQ}{d\rho}\right|_L > 0$ $\left.\frac{dQ}{d\rho}\right|_R > 0$

$\left.\frac{dQ}{d\rho}\right|_L > 0 > \left.\frac{dQ}{d\rho}\right|_R$



$\rho_L < \rho_R$ $\quad \frac{\Delta Q}{\Delta \rho} = \frac{Q_R - Q_L}{\rho_R - \rho_L} = \frac{+ve}{+ve} > 0$

min$(Q(\rho))$ satisfied.

Imp:

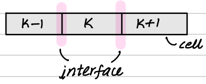
Case 1 : $\rho_L > \rho_R$	$Q = \max Q(\rho)$	$\rho_R \leq \rho \leq \rho_L$
Case 2 : $\rho_L < \rho_R$	$Q = \min Q(\rho)$	$\rho_L \leq \rho \leq \rho_R$

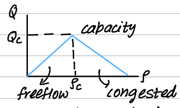
$$\tilde{\rho}_i^{n+1} = \tilde{\rho}_i^{n} + \frac{\Delta t}{\Delta x}(Q_{in} - Q_{out})$$

later generalize to minimum.

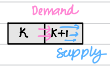
Q_{in} & Q_{out}
find from case 1 & case 2

Cell Transmission Model





(Fundamental Diagram)

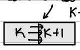


single lane

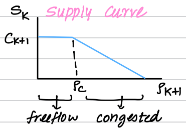
supply control — traffic information travels upstream
demand control — traffic information travels downstream

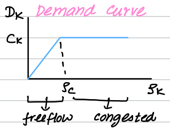
Supply - how much you can accomodate

Demand — how much veh can send when K+1 empty.



road geometry





Supply and Demand at cell boundaries

- For downstream boundaries,

$$S_{K+1}(t) = \begin{cases} Q_{K+1}^{tot}(t) & ; \ \rho_{K+1}(t) > \rho_c, \text{ cell } K+1 \text{ congested} \\ C_{K+1} & ; \text{ otherwise} \end{cases}$$

$$D_K = \begin{cases} Q_K^{tot} & ; \ \rho_{K+1} \leq \rho_c, \text{ free traffic in cell } K \\ C_K & ; \text{ otherwise} \end{cases}$$

- For upstream boundaries,

S_K, D_{K-1} } replace K by $K-1$ in above eqns

Flow through the cell boundaries

supply — how much you can accomodate
demand — assume K+1 empty

- $Q_K^{up} = Q_{K-1}^{down} = \min(S_K, D_{K-1})$
- $Q_K^{down} = Q_{K+1} = \min(S_{K+1}, D_K)$

Stability Analysis — won't study it! — we go to traditional traffic engr. ↳ come in endsem!

$\boxed{\Delta t < \frac{\Delta x}{\max|c|}}$ $\quad c = \frac{dQ}{d\rho} \quad \rho \in (0, \rho_{max})$

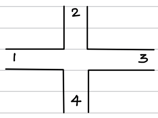
Δx varies from 100 to 150m

(Section 8.5.7.1 Trieber, Pg105) Cell Transmission Model

SIGNALISED INTERSECTION — TRAFFIC SIGNAL

16 Oct

Intersection

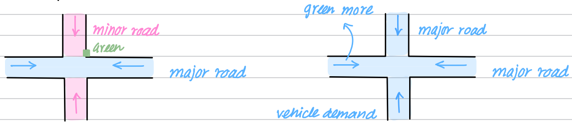


vehicles → phase
→ space

· safety
· Efficiency

Control strategies ⟶ traffic signal

Traffic Signal
- Pre-timed signal → fix the time, time is fixed
- Semi-automated signal
- Fully-automated signal



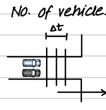
1. arrival process
2. departure process
3. delay
4. queue length

$\left\{ \begin{array}{ll} C_{12} > 0 & \text{downstream} \\ C_{12} < 0 & \text{upstream} \end{array} \right\}$

where C_{12} = propogation velocity

Arrival process

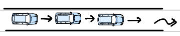
No. of vehicles arriving during specified period of time.



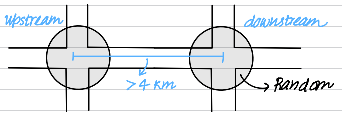
No. of vehicles in this Δt time period.

1. Random Arrival
2. Grouped Arrival
3. Mixed Arrival

Random Arrival

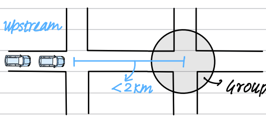


If purely random, arrival follows poisson distribution
headway follows exponential distribution.



Grouped Arrival

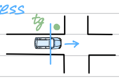
platoons — grouped vehicles
arrival in groups/platoons
headway — uniform



Mixed Arrival

some vehicles — random arrival
other " — grouped "
} mix of both
2-4 km away from another upstream

Departure Process

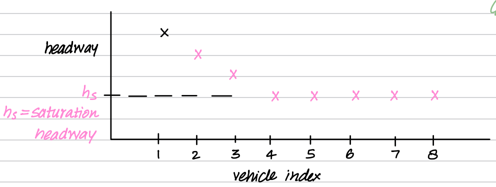


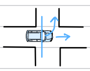
Vehicle — departures from stop line
headway of vehicles.

Initial vehicles take longer time because of :-
1. perception reaction time
2. acceleration time ~ accn needs some time

t_g — signal turns green
t_1 — 1st vehicle crosses stop line
headway $= h_1 = t_1 - t_g$
$h_2 = t_2 - t_1$
$h_3 = t_3 - t_2$
$h_4 = t_4 - t_3$ and so on . . .

Green
↓
Yellow/Amber ⟹ useful for crossing & comfortable stopping.
↓
Red
↓
Green again





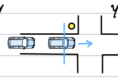
start up lost time

$l_s = \sum_{i=1}^{4} (h_i - h_s)$ } summation of headway of all the 4 vehicles

we assume after 4th vehicle, there is no delay.

movement lost time

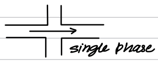
yellow period — what kind of losses?
$t = t_g$ vehicle sees yellow
$t = t_g + 5$

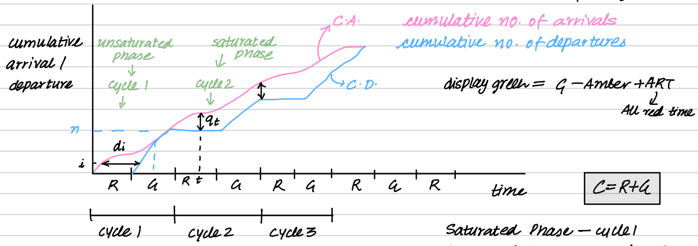


Saturation Flow
$Q_s = \frac{1}{h_s}$

startup lost time — during green phase
movement lost time — during yellow phase

DELAY AND QUEUE ANALYSIS





cumulative no. of arrivals
cumulative no. of departures

display green $= G -$ Amber $+$ ART (All red time)

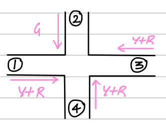
$\boxed{C = R + G}$

Saturated Phase — cycle 1
Unsaturated Phase — cycle 2, 3 . .
oversaturated

d_i = delay of i^{th} vehicle
q_t = no. of vehicles in the Queue at time t
R = effective red time = time during which no vehicle on that lane cross the intersection.
G = effective green time = time during vehicles on that lane cross the intersection.

C (cycle time) = green to green time = R+G = constant (don't change)
G (effective green time) = Green time − (startup lost time + movement lost time)
R (effective red time) = Cycle time − Eff. green time

- unsaturated ($\lambda C < Q_s G$): total no. of arrivals ≤ total no. of vehicles that can be served by the system
- saturated ($\lambda C \geq Q_s G$): total no. of arrivals > total no. of vehicles that can be served by the system — oversaturated



Yellow = 4 to 5 seconds
$t_1 = g$
All the approaches, cycle time is same.
Based on volume, red time & green time changes.

Delay Analysis (Unsaturated System)

Assumptions
1. Deterministic arrival process
2. Uniform arrival rate (λ vehicles per unit time) eg:- 10 veh/min
3. Unsaturated system

cycle time = C
No. of vehicles arriving during cycle time 'c' = λc $\quad \left\{ \frac{veh}{sec} \times sec \right\}$

Departure Rate

$\alpha_a = 0$ — during red time

$\alpha_g = s$ (saturation flow rate) — during green time

$Q_s = \frac{1}{h_s} = s$ (saturation flow rate)

effective green time $(G) = g - (\text{all } \cancel{\text{losses}}) = G$

vehicles departure during eff. green time $= \alpha G = sG$

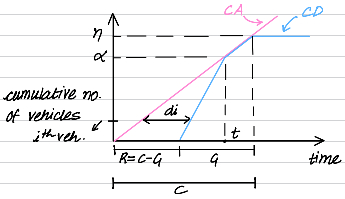
unsaturated phase / condition $\quad vc \leq sG \quad$ (bcoz of overflow from last phase)

saturated phase / condition $\quad vc > sG$

Assumptions

19 Oct

1. deterministic arrival
2. uniform arrival rate 'v' veh / time
3. unsaturated system



cumulative arrival
cumulative departure
d_i = delay of i^{th} vehicle
$C = R + G$ = cycle time

Total Delay $\quad TD = \sum_{i=1}^{n} d_i$

TD = area under the curve $= \frac{1}{2} \times b \times h$

$b = (c - g)$ $\quad h = \alpha$

cycle time, eff. green time

During time 't'

cumulative arrival $= vt = \alpha$

cumulative departure $= \hat{s}(t - (c-g)) = \alpha$

$vt = \hat{s}(t - (c-g))$

$vt = \hat{s}t - \hat{s}(c-g)$

$t(\hat{s} - v) = \hat{s}(c-g)$

$$\boxed{t = \frac{\hat{s}(c-g)}{(\hat{s}-v)}}$$

$\alpha = vt = \frac{v \times \hat{s}(c-g)}{(\hat{s}-v)}$ $\quad \left\{ \frac{g}{c} = \text{effective green ratio} \right\}$

$TD = \frac{1}{2} \times \text{base} \times \text{height} = \text{Area of } \Delta =$

$= \frac{1}{2} \times (c-g) \times \frac{v\hat{s}(c-g)}{(\hat{s}-v)} = \frac{v\hat{s}(c-g)^2}{2(\hat{s}-v)} = \frac{v(c-g)^2}{2(1-v/\hat{s})}$

Average Delay $\quad AD = \frac{\text{Total Delay }(TD)}{\text{Total no. of vehicles}}$

No. of vehicles $= vc$

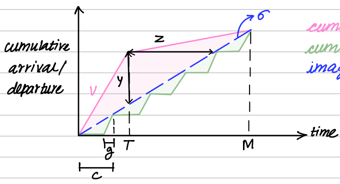
$AD = \frac{\frac{v(c-g)^2}{2(1-v/\hat{s})}}{vc} = \frac{v(c-g)^2}{2(1-v/\hat{s})vc} = \frac{c(1-g/c)^2}{2(1-v/\hat{s})}$

$$\boxed{AD_{us} = \frac{c(1-g/c)^2}{2(1-v/\hat{s})}}$$ (unsaturated condition)

Delay Analysis (oversaturated condition)

Assumptions

1. Deterministic arrival process
2. Uniform arrival rate from 0 to T ('v' veh/time)
3. saturated condition (v is large enough to cause oversaturation) $vc > sg$



cumulative arrival
cumulative departure
imaginary line for unsaturated condition → slope = σ

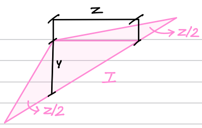
slope of dotted line $= \sigma$

No. of vehicles arriving during cycle period 'c' $= \sigma c$

Vehicles departure during green time $g = sg$

$\sigma c = sg \quad$ or $\quad \sigma = \left(\frac{s}{c}\right) g$

Average Delay of Area I (Pink color)



$AD_I = \frac{z/2 + z/2}{2} = \frac{z}{2}$

Average Delay of Area II (Green color)

$AD_{II} = \frac{1}{2} \frac{c(1-g/c)^2}{(1-v/s)} = \frac{1}{2} \frac{c(1-g/c)^2}{(1-\sigma/s)}$

$= \frac{1}{2} \frac{c(1-g/c)^2}{\left(1 - \frac{\frac{s}{c} \times g}{s}\right)} = \frac{1}{2} \frac{c(1-g/c)^2}{(1-g/c)}$

$= \frac{(c-g)}{2}$

Average Delay for oversaturation condition

$AD_{os} = AD$ of area I $+ AD$ of area II $= AD_I + AD_{II}$

$AD_{os} = \frac{z}{2} + \frac{c-g}{2}$

$\sigma = \frac{y}{z} \qquad y = vT - \sigma T \qquad \sigma c = s g$ (saturation flow, eff. green time) $\qquad \sigma = \left(\frac{s}{c}\right) g$

$z = \frac{y}{\sigma} = \frac{vT - \sigma T}{\sigma} = T\left(\frac{v-\sigma}{\sigma}\right)$

Put $\sigma = \left(\frac{s}{c}\right) g$

$z = T\left(\frac{v - sg/c}{sg/c}\right) = T\left(\frac{v}{sg/c} - 1\right) = T\left(\frac{v}{\hat{c}} - 1\right) \qquad \left\{ \text{where } \hat{c} = \frac{sg}{c} \right\}$

Put in AD_I formula

$AD_I = \frac{z}{2} = \frac{T}{2}\left(\frac{v}{\hat{c}} - 1\right)$

Average Delay for over-saturated condition

$$\boxed{AD_{os} = \frac{T}{2}\left(\frac{v}{\hat{c}} - 1\right) + \left(\frac{c-g}{2}\right)}$$ (oversaturated condition)

In reality, however arrival is not deterministic, it is stochastic. For stochastic arrival above relations cannot be used and at best can give approximate estimates. For real field, you'll need to apply some correction factors.

Webster Empirical Relations to determine delay

Assumptions

Arrivals follow poisson distribution $\qquad$ average arrival rate $= v$

Departures occur uniformly $\qquad$ max departure rate $= s$

condition : $vc \leq gs$

$D_{web} = \underbrace{\frac{c(1-g/c)^2}{2(1-v/s)}}_{\text{uniform arrival}} + \underbrace{\frac{(v/c)^2}{2v(1-v/c)}}_{\text{stochastic traffic flow nature}} - \underbrace{0.65\left(\frac{c}{v^2}\right)^{1/3}\left(\frac{v}{c}\right)^{2+(5g/c)}}_{\text{correction factor}}$

$D_{web} = 0.9 \times \left[\frac{c(1-g/c)^2}{2(1-v/s)} + \frac{(v/c)^2}{2v(1-v/c)}\right] \qquad \frac{v}{c} \to 1$ term explodes.

$\frac{v}{\hat{c}} = 1 \qquad$ Put $\hat{c} = \frac{s}{c} g \Rightarrow \frac{v}{(sg/c)} = 1 \Rightarrow \boxed{vc = sg}$

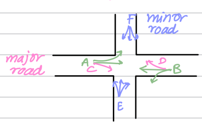
Highway Capacity Manual 1985 Delay Relation

$D_{HCM} = 0.76 \frac{c(1-g/c)^2}{2(1-v/s)} + 173\left(\frac{v}{c}\right)^2\left[\left(\frac{v}{c}+1\right) + \sqrt{\left(\frac{v}{c}-1\right)^2 + \frac{16v}{c^2}}\right]$

Signal Timing Design of Signalised Intersections

1) phasing scheme
2) cycle length
3) phase lengths

Phasing Scheme



phases ↑es ⇒ losses also ↑es

⇒ less time for vehicle movement ⇒ higher average delay

Thus, more no. of phases ⇒ greater safety but lower efficiency

Generally 2 to 6 phases are kept to trade up b/w efficiency and conflicts.

phase I → A & B } green + Amber + Red — same time $\qquad$ phase I ①

phase II → C & D $\quad$ green + Amber + Red — same time $\qquad$ phase II ③

phase III → E & F $\qquad$ phase III ② $\qquad$ phase IV ④

(phase schemes can be different)

Signalised Intersection Contd....

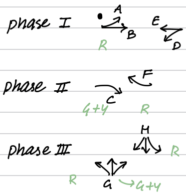
30 Oct

Three signals → cyclic manner
G, Y, R (Green, Yellow, Red)

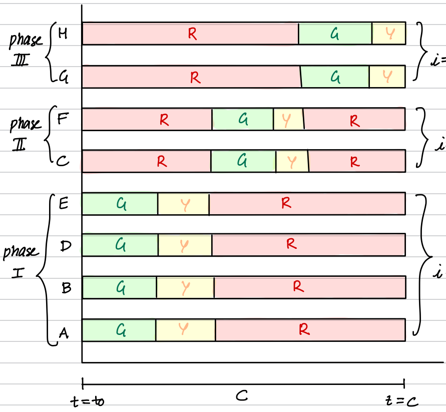
1) phase scheme
2) cycle scheme
3) phase length

Phase scheme

phase ↑, delay ↑, losses ↑
startup lost time
movement lost time } conflicts
loss no. of phases



$t = t_0$ starting time of green
$t = t_1$ starting time of green to next cycle (G+Y)
cycle time $= t_1 - t$



$i=3$ $\left(\frac{V}{S}\right)_{cr,3} = \max\left(\frac{V_G}{S_G}, \frac{V_H}{S_H}\right)$

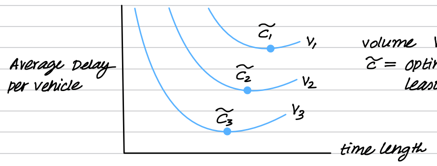
$i=2$ $\left(\frac{V}{S}\right)_{cr,2} = \max\left(\frac{V_C}{S_C}, \frac{V_F}{S_F}\right)$

$i=1$ (V_E/S_E), (V_D/S_D), (V_B/S_B), (V_A/S_A) — critical V/s of phase I is max. of these four.

$$\left(\frac{V}{S}\right)_{cr,1} = \max\left(\frac{V_A}{S_A}, \frac{V_B}{S_B}, \frac{V_D}{S_D}, \frac{V_E}{S_E}\right)$$

cycle length

primary concern → average delay of vehicles



volume $V_1 > V_2 > V_3$
$\tilde{c}$ = optimum cycle time / least delay point

Observations from this graph :-

- Nature of effect on delay is not always monotonic (strictly ↑ or ↓ function). It first ↓ and then ↑ so there exist a least delay point.
- $\tilde{c} = 62.1824 \approx 65$ (Kept multiple of 5 generally)
  Roundoff → go to right side of the optimum cycle time since the sensitivity of variation (slope) is less on right as compared to left side.

$$\boxed{C_0 = \frac{1.5L + 5}{1 - \sum_{i=1}^{P}\left(\frac{V}{S}\right)_{cr_i}}}$$

eqn to determine optimum cycle length C_0 given by webster (IRC 93:1985 also suggest this)

L = total time lost per cycle (includes all the phases) in sec
p = total no. of phases in the cycle
V = volume of a particular movement for phase i
S = saturation flow for the movement for phase i
$(V/S)^i_{cr}$ = critical flow ratio for phase i

$\left(\frac{V}{S}\right)^i_{cr} = \max\left(\frac{V_1}{S_1}, \frac{V_2}{S_2}, \frac{V_3}{S_3}, \frac{V_4}{S_4}, \ldots\right)$ maximum of all flow ratios obtained for that phase

$S = 525W$, W = width of lane

L = total loss time $= \sum_{i=1}^{P}\left(\ell^i_s + \ell^i_m + \ell^i_r\right)$

start up lost time, movement lost time, all red time

★ If $\frac{V}{S} \geq 0.975$ method wont work, $C_0 = -ve$ (−ve cycle length not possible) — heavy/congested traffic

Phase length

for safety all phases have red for some time as amber of one ends.

Green time

$G_1 = C_0 \times \left(\frac{V_1}{S_1}\right)_{cr}$

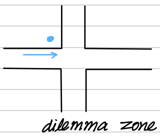
$G_2 = C_0 \times \left(\frac{V_2}{S_2}\right)_{cr}$

$G_3 = C_0 \times \left(\frac{V_3}{S_3}\right)_{cr}$

multiply optimum cycle length by critical flow for that phase to calc. green time.

Yellow/Amber Time through Dilemma zone Analysis

Amber → two choices
1. stop
2. cross the intersection

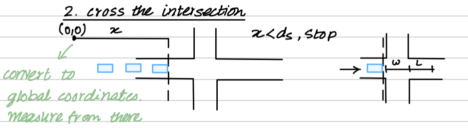


1. stop

$$d_s = V t_r + \frac{V^2}{2b}$$

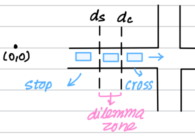
V → design speed; d_s → stopping distance; t_r → reaction time ↳ to recognize traffic light change to yellow; b → comfortable deceleration rate

2. cross the intersection



L = length of vehicle
w = width of intersection
τ
$d_c < 0$ or > 0

$d_c = V\tau - (w + L)$; τ → amber time ; d_c → distance travelled during amber time



$x > d_c$ — cross the intersection
$x < d_s$ — stop
$d_s < x < d_c$ — dilemma zone (neither cross nor stop)

The minimum amber time to remove dilemma zone (τ_{min})
put $d_s = d_c$ to avoid dilemma zone

$V\tau_{min} - (w+L) = V t_r + \frac{V^2}{2b}$ ↳ Put with sign (−ve) → $v^2 - u^2 = 2as$, $0 - u^2 = 2as$, $s = \frac{-v^2}{2a}$ | $s = +ve$, $a = -ve$

$\tau_{min} = \frac{V t_r}{V} + \frac{V^2}{2bV} + \frac{(w+L)}{V}$

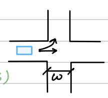
$$\boxed{\tau_{min} = t_r + \frac{V}{2b} + \frac{(w+L)}{V}}$$

Pedestrian Crossing Time

pedestrians cross roads as a group

$t_p = 7 + \frac{w}{1.2}$

7 → pedestrian reaction time (7sec); w → width of the intersection (m); 1.2 → average pedestrian speed (1.2 m/s) (IRC:93-1985)



TRAFFIC SIGNS

1. speed limit
2. inform on impending changes in road geometry
3. reduce confusion thro' clear sign.

Design Elements of Traffic Sign

1. text of the sign
2. letter and color of sign
3. placement of the sign

traffic sign: 1. visual acuity — 1. words ×; 2. field of vision — 2. size of letter; 3. colour perception — 3. placement

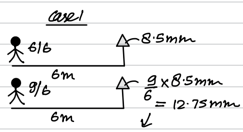
Text of the sign

IRC 67–1977
In general, dont use text & use shapes / pictograms (brief text only)

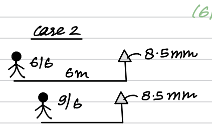
Letter and Color of the sign

- normal vision driver → 8.5mm (letter size) from 6m distance (6/6)
- poor vision driver → $\left(\frac{6}{9}\right)$ Two cases: $\left(\frac{6}{9}\right)$ & $\left(\frac{6}{12}\right)$

normal driver (6/6), (6/9) case 1, (6/12) case 2



In general, $\frac{9}{6} \times h$ ↳ letter size (mm)

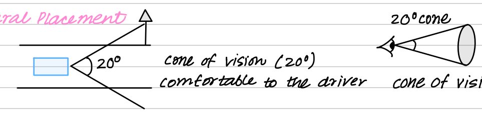


$\left(\frac{6}{9}\right) \times 6 = 4\,m$

In general, $\frac{6}{9} \times d$ ↳ distance (m)

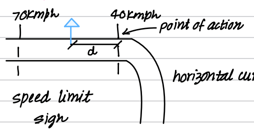
2 Nov

Lateral Placement



cone of vision (20°)
comfortable to the driver
cone of vision

Longitudinal Placement

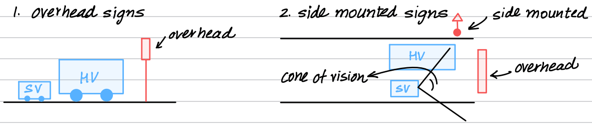


d = longitudinal placement

Occlusion

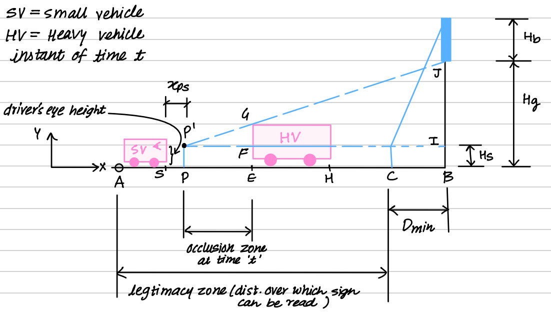
road signs can be occluded (hidden) in two cases

1. overhead signs
2. side mounted signs



overhead signs

SV = small vehicle
HV = Heavy vehicle
instant of time t



Assume from point A, driver seeing the sign board

A = starting point (0,0)
B = $(D_{max}, 0)$
C = $(D_{max} - D_{min}, 0)$ → upto pt C sign can be read
S = $(x_s(t), 0)$
P = $(x_p(t), 0)$
H = $(x_H(t), 0)$
AC = legtimacy zone at time t
PE = occlusion zone at time t

Assumptions
1. At time $t=0$, SV $x_s(0) = 0$ → point A
2. SV traveling constant speed throughout the domain $[0, D_{max}]$

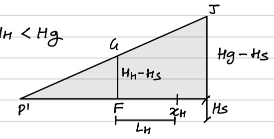
$x_{ps} = x_p - x_s$
$x_s = x_s(0) + u_s t = u_s t$
$x_s = u_s t$ ——— (1)

x_p

$$\left[\frac{H_H - H_s}{H_g - H_s}\right] = \frac{x_H - L_H - x_p(t)}{D_{max} - x_p(t)}$$

constant $= \alpha \in (0,1)$

$H_s < H_H < H_g$



(similar Δ property)

$x_H - L_H - x_p(t) = \alpha(D_{max} - x_p(t))$
$x_H - L_H - \alpha D_{max} = x_p(t)[1-\alpha]$

$$x_p(t) = \frac{x_H - L_H - \alpha D_{max}}{1-\alpha}$$

Assumptions
1. $t=0$ HV → lies b/w $[0, C]$ $x_H(0) = d_0$
2. speed = constant (u_H)
$x_H(t) = x_H(0) + u_H t$
$x_H(t) = d_0 + u_H t$ → Put in above 'x_p' formula

$$x_p(t) = \frac{u_H t + d_0 - L_H - \alpha D_{max}}{1-\alpha}$$

$$x_p(t) = \frac{u_H}{1-\alpha} t + \underbrace{\frac{d_0 - L_H - \alpha D_{max}}{1-\alpha}}_{= \text{constant} = x_p(0)}$$

$$x_p(t) = \frac{u_H}{1-\alpha} t + x_p(0)$$

$x_p(t) = u_p(t) t + x_p(0)$
$x'_p(t) = \frac{u_H}{1-\alpha} = u_p(t)$

$u_p(t) = \frac{u_H}{1-\alpha}$ ⇒ $u_H = (1-\alpha) u_p$ Put above

$x_p(t) = \frac{1}{1-\alpha} \times (1-\alpha) u_p t + x_p(0)$

$x_p(t) = u_p t + x_p(0)$ ——— (2)
$x_s(t) = u_s t$ (from eqn 1)

$x_{ps}(t) = x_p(t) - x_s(t)$
$= u_p t + x_p(0) - u_s t$
$= (u_p - u_s) t + x_p(0)$

$$\boxed{x_{ps}(t) = (u_p - u_s) t + x_p(0)}$$

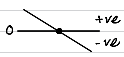
$x_{ps} < 0$ Occlusion
$x_{ps} \geq 0$ No occlusion

CASES FOR OCCLUSION

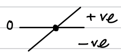
$[A, C]$
$[0, T]$
$T = \frac{D_{max} - D_m}{u_s}$

Case 1: $u_p \geq u_s$, $x_p(0) \geq 0$
$x_{ps}(t) = (+ve) + (+ve) = +ve$
Not occluded

Case 2: $u_p < u_s$, $x_p(0) \geq 0$
$x_{ps}(t) = (-ve) + (+ve)$
Initially not occluded.
After some point of time, occlusion happens.



Case 3: $u_p \geq u_s$ $x_p < 0$
$x_{ps}(t) = (+ve) + (-ve)$
Initially occlusion.
After some point of time, not occluded.



Case 4: $u_p < u_s$ $x_p < 0$
$x_{ps}(t) = (-ve) + (-ve)$
Always occluded.

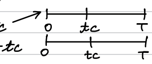
case 2 not occluded / occluded
case 4 occluded / not occluded } → Put $x_{ps} = 0$

$x_{ps}(t) = (u_p - u_s) t + x_p(0)$
$(u_s - u_p) t = x_p(0)$

$$\boxed{t_c = \frac{x_p(0)}{u_s - u_p}}$$

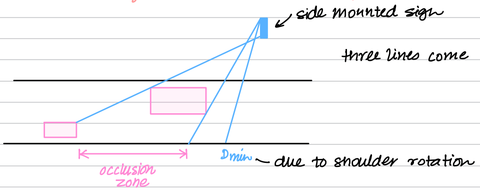
t_r = time required to read the sign
t_a = time available to read the sign

Case 1 $t_a = T$
Case 2 $t_a = t_c$
Case 3 $t_a = T - t_c$
Case 4 $t_a = 0$



$t_a \geq t_r$ → read the sign.
$t_a < t_r$ → not possible to read the sign.

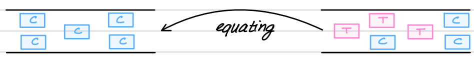
For side mounted signs (won't come in Exam)



Passenger car Equivalents

9 Nov

factor



Different traffic stream

Traffic Stream characteristics
1. speed v
2. flow Q
3. density ρ

Factors
1. % of a particular vehicle type (i) in traffic stream → HV
2. geometry of the road

These two factors (% + geometry) affect the traffic stream characteristics (v, Q, ρ)
Equivalents — equate different traffic streams based on their characteristics.

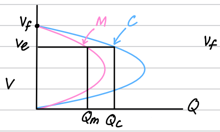
1) equal speed
2) equal density
3) equal delay (intersection)
4) equal headway

} PCE criterias

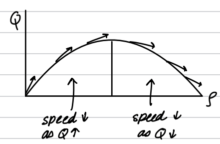
equal speed

1) car only traffic stream (C)
2) mixed traffic stream (M) → C + HV

assumed same speed of car and heavy vehicle



V_f = max speed



M: P → proportion of HV
$1-P$ → proportion of car
PCE_i $(i = HV)$

$$\underbrace{(1-P) Q_m}_{car} + \underbrace{PCE_i \, P Q_m}_{car} = \underbrace{Q_c}_{\text{car only traffic stream}}$$

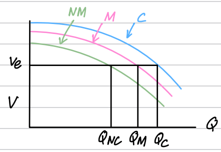
mixed traffic stream

$Q_m - PQ_m + PCE_i\, PQ_m = Q_c$
$PCE_i PQ_m - PQ_m = Q_c - Q_m$
$PQ_m\,(PCE_i - 1) = Q_c - Q_m$
$PCE_i - 1 = \left(\frac{Q_c - Q_m}{Q_m P}\right)$

$$\boxed{PCE_i = 1 + \frac{1}{P}\left(\frac{Q_c}{Q_m} - 1\right)}$$

Now three traffic streams are there :-
1) car only traffic stream (C)
2) mixed traffic stream (M) → C + HV
3) new mixed traffic stream (NM)

ΔP → New Mixed → New vehicle type j
P → i → existing vehicle type (M)
$(1-P-\Delta P)$ → car (base stream)



P → vehicle type i
ΔP → " " j } 3 vehicle type
$1-P-\Delta P$ → car
PCE_j

$$\underbrace{(1-P-\Delta P)Q_{NM}}_{c} + \underbrace{PCE_i\, PQ_{NM}}_{c} + \underbrace{PCE_j\,\Delta P Q_{NM}}_{c} = \underbrace{Q_c}_{\text{car only traffic stream}}$$

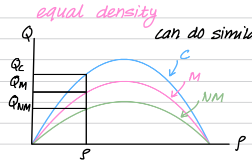
$Q_{NM} - PQ_{NM} - \Delta PQ_{NM} + PCE_i PQ_{NM} + PCE_j \Delta P Q_{NM} = Q_c$
$(PCE_i - 1)PQ_{NM} + (PCE_j - 1)\Delta P Q_{NM} = Q_c - Q_{NM}$
$(PCE_i - 1)P + (PCE_j - 1)\Delta P = \left(\frac{Q_c}{Q_{NM}} - 1\right)$

from 2 traffic stream $= \frac{Q_c}{Q_m} - 1$ →
$\frac{Q_c}{Q_M} - 1 + (PCE_j - 1)\,\Delta P = \left(\frac{Q_c}{Q_{NM}} - 1\right)$

$(PCE_j - 1)\Delta P = \frac{Q_c}{Q_{NM}} - \frac{Q_c}{Q_M}$

$$\boxed{PCE_j = 1 + \frac{1}{\Delta P}\left(\frac{Q_c}{Q_{NM}} - \frac{Q_c}{Q_M}\right)}$$

equal density



can do similarly . . . .
disadvantages
equal speed &
equal density

car only traffic stream graph
{ simulation methods
theoretical models
v − Q } graph
K − Q } — not easy

headway method

$$\boxed{PCE_i = \frac{\bar{h}_i}{\bar{h}_c}}$$

$\bar{h}_i$ = average time headway of vehicle type i
$\bar{h}_c$ = average time headway of car (base vehicle)

- For two wheelers $PCE_{TW} = \frac{\bar{h}}{\bar{h}_c}$ → smaller → larger $\quad \bar{h}_{TW} < \bar{h}_c$

$PCE_{TW} < 1$
Generally, $PCE_{TW} = 0.5 \Rightarrow$ 2 two wheelers equivalent to 1 car

- For cars $PCE_c = \frac{\bar{h}_c}{\bar{h}_c} = 1$

- For heavy vehicles $PCE_{HV} = \frac{\bar{h}_{HV}}{\bar{h}_c}$ → larger → smaller $\quad \bar{h}_{HV} > \bar{h}_c$

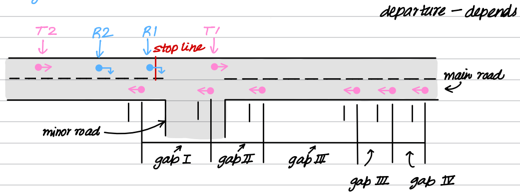
$PCE_{HV} > 1$
Generally, $PCE_{truck} = 2.5 \Rightarrow$ 1 Truck = 2.5 car

- $PCE_{TW} < PCE_c < PCE_{HV}$
$PCE_{TW} < 1 < PCE_{HV}$

Unsignalized Intersections

Unsignalised intersection

arrival — same as signalized int.
departure — depends on hierarchical position



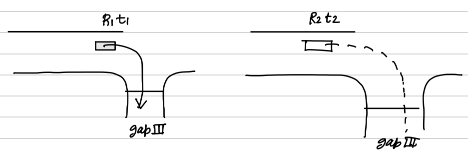
departure and arrival process from stop line
T_1 → reaching time $\bar{t}$ = arrival time = departure time
R_2 → Right turning vehicle
Wait for adequate gap
gap I small → no action
gap II small → no action
gap III large → make right turn
→ critical gap (t_c) → min. adequate gap
or
min. required gap

gap III ≥ t_c
observed quantity → latent variables

gap I, gap II < t_c

NOTE: Always critical gap is time gap (t_c) and not the distance gap
(t_c) → time gap

gap III large



Follow up time, $t_f = t_2 - t_1$
Time Difference b/w two successive vehicles (stop line)
↳ same observer g_a

min. required time (t_c) → it is a random variable
(not a deterministic variable)
→ it is continuous random variable
(not discrete random variable)

$t_c \sim$ follows log normal distribution usually
$\ln(\mu, \sigma^2)$
fit the observed data to the distribution

maximum likelihood estimation — maximize your prob. using observed data

13 Nov

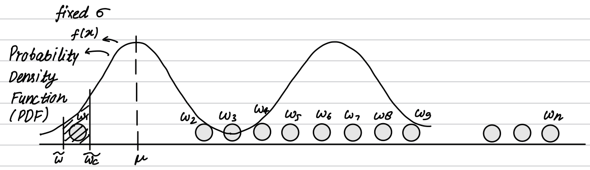
t_c → critical gap
↳ Not deterministic
↳ Continuous Random Variable (μ, σ^2) — mean, variance

Maximum Likelihood Estimation (MLE)

Find out the optimum parameter (μ, σ^2) using observed data.

Normal, Exponential, lognormal

fixed σ



∵ Continuous Random Variable
$P(X = w_1) = 0 = P(X = w_2) = \cdots$
$P(\tilde{w} < X < \tilde{w}_c)$ = Area under the curve

$P = \int \underbrace{f(x)}_{PDF} dx \qquad P = f \quad (\because \Delta x \to 0)$

A → R.V. } $P(A \cup B) = P(A) \times P(B)$
B → R.V.

Likelihood : $(L) = f(x)$
total probability

X — Random Variable
x — observed value

$x \rightarrow i.i.d. \rightarrow$ independent and identically distributed

$x_1 \rightarrow \omega_1$ N

$x_2 \rightarrow \omega_2$ N

$x_3 \rightarrow \omega_3$ N

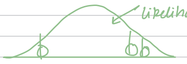
$L = P(X_1=\omega_1) \times P(X_2=\omega_2) \times P(X_3=\omega_3) \times \cdots \times P(X_n=\omega_n)$

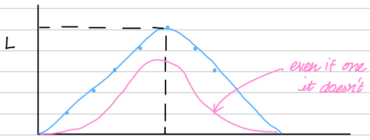
$L = \prod_{i=1}^{n} P(x_i=\omega_i)$

sometime pdf
" cdf

$\underset{\uparrow}{L} = \prod_{i=1}^{n} f(x_i=\omega_i)$

likelihood



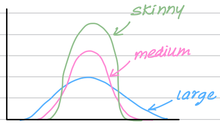


position of height of the distribution

$L = \prod_{i=1}^{n} f(x - x_i(\mu))$

$\frac{\partial L}{\partial t} = 0$

- Discrete $\rightarrow$ only probability mass functions
- Continuous $\rightarrow$ Density function / $\rightarrow$ Probability function } — depends on your problem based on that you've to design



$\frac{\partial L}{\partial h} = 0$

$\frac{\partial L}{\partial \sigma} = 0$

Instead of likelihood we go to log likelihood because it is easy.

$L = \prod_{i=1}^{n} f(x_i=\omega_i)$

$LL = \sum_{i=1}^{n} f(x_i=\omega_i)$

$\log \rightarrow$ monotonically $\uparrow$sing function.

Poisson Distribution

Vehicle Arrival $\rightarrow$ poisson distribution (λ) ← mean arrival rate

$x_1, x_2, x_3, \ldots\ldots x_n$

$\downarrow n_1 \ \downarrow n_2 \ \downarrow n_3 \ \ldots \ \downarrow n_n$

Poisson $\rightarrow$ Discrete Distribution

$P(X_1=n_1) = \frac{e^{-\lambda}\lambda^{x_1}}{x_1!}$

$P(X_2=n_2) = \frac{e^{-\lambda}\lambda^{x_2}}{x_2!}$

$P(X_n=n_n) = \frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$

$L = P(X_1=x_1) \times P(X_2=x_2) \times P(X_3=x_3) \times \cdots \times P(X_n=x_n)$

$= \frac{e^{-\lambda}\lambda^{x_1}}{x_1!} \cdot \frac{e^{-\lambda}\lambda^{x_2}}{x_2!} \cdot \ldots \ldots \frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$

$L = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$

$LL = \ln\left(\prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right)$

$= \sum_{i=1}^{n} \left[\ln(e^{-\lambda}) + \ln(\lambda^{x_i}) - \ln(x_i!)\right]$

$= \sum_{i=1}^{n}(-\lambda) + \sum_{i=1}^{n} x_i \ln(\lambda) - \sum_{i=1}^{n} \ln(x_i!)$

$LL = -n\lambda + \ln(\lambda)\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \ln(x_i!)$

$\frac{dLL}{d\lambda} = -n + \frac{\sum x_i}{\lambda} - 0$

$\frac{dLL}{d\lambda} = 0 \qquad -n + \frac{\sum x_i}{\lambda} = 0$

biased estimator
unbiased estimator

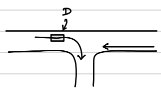
$\lambda = \left(\frac{\sum x_i}{n}\right) \qquad n = \frac{\sum x_i}{\lambda}$

$t_c \rightarrow$ continuous RV

1) driver $\rightarrow$ consistent

accepted gap $\rightarrow$ one

rejected gap $\rightarrow$ many



rejected gaps(sec)   accepted gaps(sec)

5, 3, 8, 4 } many ↳ max = 8 sec

10 } only one gap

Assume always Accepted gaps > Rejected gaps

critical gap (t_c)

$t_c >$ Rejected gap (R_n) } $\quad R_n < t_c \leq A_n$

$t_c \leq$ Accepted gap (A_n)

$P(R_n < t_c \leq A_n)$
$= P(R_n \leq t_c \leq A_n)$ {single pt prob. = 0}

$t_c \sim N(\mu, \sigma^2)$ Normal Distribution

$t_c \rightarrow iid$

$P(R_n \leq t_c \leq A_n)$

$n \rightarrow$ observations

$x_1 \downarrow A_1, R_1 \quad x_2 \downarrow A_2, R_2 \quad \ldots \quad x_n \downarrow A_n, R_n$

$L = \prod_{i=1}^{n} P(R_i \leq t_c \leq A_i)$

$\frac{\partial LL}{\partial \mu} = 0 \qquad \frac{\partial LL}{\partial \sigma} = 0$

Follow rules, Ethics and Integrity...