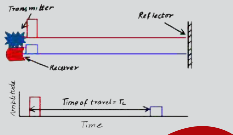
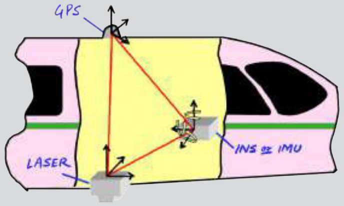
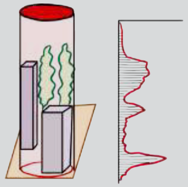


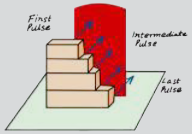


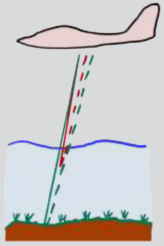
CE676A



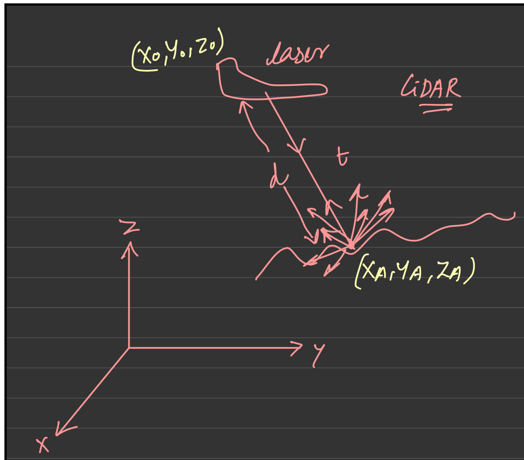
Laser Scanning & Photogrammetry

Prof. Bharat Lohani



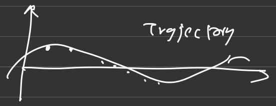


Aman



Lecture-2

- DGPS — 2cm
  interpolation
  IMU — measures accⁿ
  ↳ orientation of the plane



last return can be from any point.

spectral wavelength $\rho_\lambda = \frac{E_{R(\lambda)}}{E_{I(\lambda)}}$

Radiometric Resolution

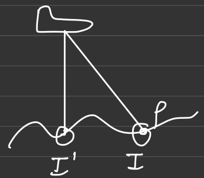
(x, y, z, I)

$(2^n - 1)$ $0 - 255$

Teledyne → Multispectral Lidar

FCC → Remote Sensing.

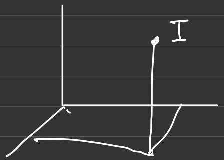
Classified Data in Lidar



I – how much we're getting back

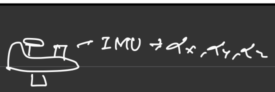
$(2^n - 1)$, n → quantisation wt

0-255



Color scale → by I, height, etc.

ALS – Aerial Laser scanning
MLS – Mobile Laser Scanning

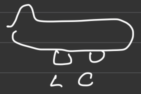


←1s→

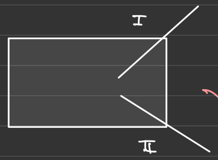
SoI → CORS
Continuously
Operating
Ref. Stations.

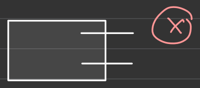

need to know trajectory coordinate

CORS → ground ref. not needed.



→ Lidar and Camera are always kept. (mostly)

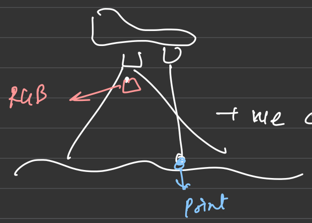




It will cover all information front & back as well.

TLS → Hor. angle
Vert. angle
→ 3 scanner → ILRIS - 3D
→ Faro
→ Reigl < 2000m, 400m

ALS
MLS
TLS
ULS → Unmanned Las. Scan
Robots.
Gadgets.



Colorisation of the Point Cloud.

→ we can associate pixel value from Cam. to point.

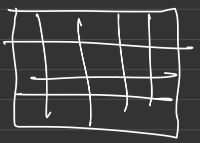
Classification → Giving classes to each point.
Telling each point who you are.

DEM → Digital Elevation Model

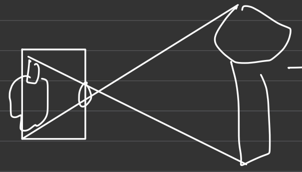
Raster grid.

Raster → we display an area by it
↓
giving it height.

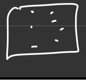
1X1 m grid



Orthophotograph → A photograph but it is ortho.



→ For camera
↳ perspective image → it has Distortions.



Scale is not uniform everywhere.

Orthophotograph → scale is uniform everywhere.

How generate? → using Lidar Data (X4.2)
→ Image

One Return.
① Single Return
② Multi Return
③ Full → maxⁿ information.

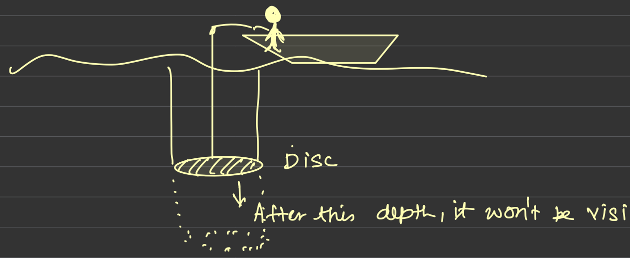
Full form → useful for forests.

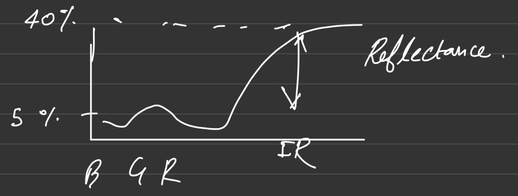
Bathymetric Data → I.R. → to capture
Green → to capture depth

↕ 3 X Secchidepth → able to observe the coordinate.

2 or 3 times secchi depth.

Secchi Depth - Depth at which if a disc is lowered into water, it will not be visible from the surface.



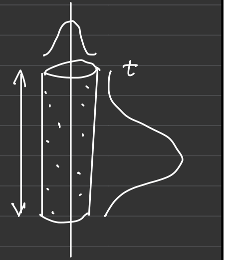


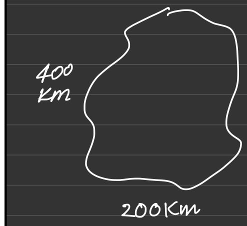
Principle of LiDAR

Ideally Lidar laser should go straight with a fixed width,

But in reality, it has got some divergence.

The cylinder of photons, laterally and horizontally have a gaussian distribution of Intensity over time.

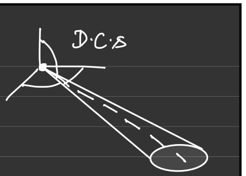




User

① Accuracy

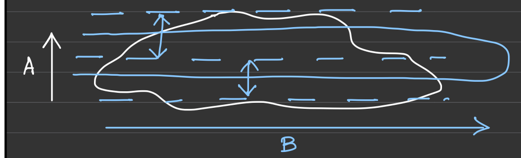
② PRMS Density



- Higher I fly, my data density ↓es and uncertainty ↑es.

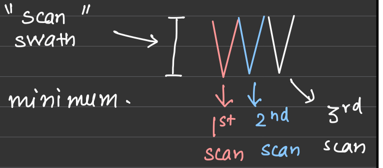
d = ?

a = ?



⇒ Here flying in linear direction will give me minimum time of travel.

'f': scan frequency



H = ? PRF = ?

V = ? f = ?

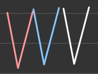
Where I fly? Decide so that time of flight is minimum.

PRF (Point Repitition Frequency): How many pulses it can fire in 1 sec.

5 lac, 8 lac,

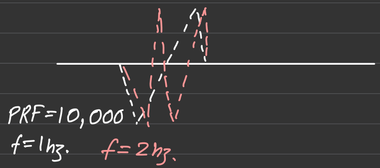
- If I fly fast then, a ⇒ no effect {∵ vel. of flight << vel. of light}
  d ⇒ decreases (↓es)

Scanning Frequency

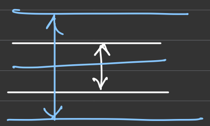


- suppose H = ✓, V = ✓, PRF = ✓ fixed and f = increased (↑) what will happen?

⇒ Distribution of the Points change.



Θ → scan angle



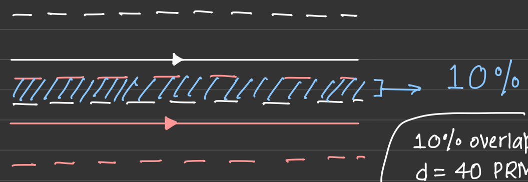
Φ
H
V
PRF
f
Θ

T — time of flight
d — density
a — accuracy

If Θ is smaller,
- Total time of travel ↑es
- Density ↑es

- Data density doesn't depend upon the direction in which I am flying.
- Accuracy → only changed by the height (altitude) of the flight.
- As height ↑es, accuracy ↓es, uncertainty ↑es.

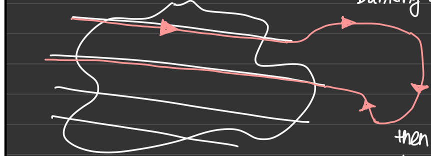
Overlap :-



user
10% overlap
d = 40 PRM
a = ----

scanning pattern : · zigzag
· sinusoidal, etc.

constant banking angle :



banking angle.
↓
If its not maintained
↓
then GPS lock breaks.

- An overlap is maintained here.
- Why this overlap is maintained ?

(1)

The path is not perfectly straight, If it displace a bit and there is no over lap then there will be some area left to be captured.

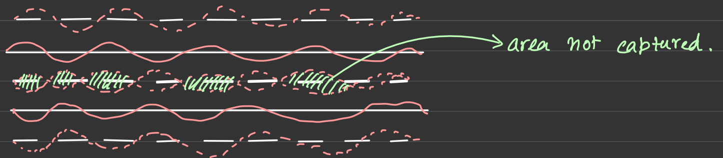
(2)

Sometimes the data collected may not be precise. Then if a overlap is there, then it helps to check the location of that data and its

---

Why an overlap is maintained ?

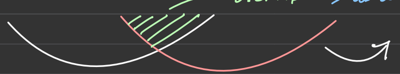
①



- The path of aircraft is not perfectly a straight line.
- If it displace a bit from straight path and there is no overlap maintained, then there will be some area left to be captured.

②

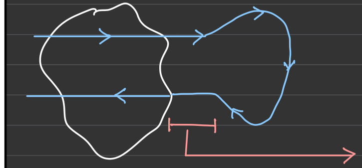
overlap → as a quality check



Data from two flight lines.

overlap — cross section — data from both the flight lines should match. Ideally that should happen, but indeed data may not match. Sometimes data collected may not be precise. Then if a overlap is there, then it helps to check the location of that data and its quality.

Cushion Period



this is called Cushion Period

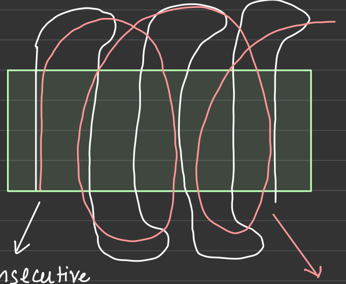
ALTM 3100EA

Specification
0-70 Hz (+1 Hz)
↓
+1 Hz change we can do in a second.

$$\boxed{T = \underbrace{T}_{\text{Total time}} \; \underbrace{T_T}_{\text{turning time}} + \sum \underbrace{T_{FL}}_{\text{flight time}} + \sum \underbrace{T_C}_{\text{cushion time}}}$$

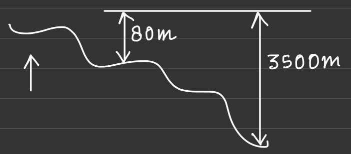
→ minimize total time.

Consecutive & non-consecutive turning :-



consecutive turning
↓
time of fligh — more

Non-consecutive turning
↓
total time of flight — less

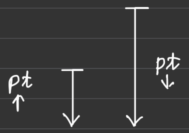


- For 80< H< max 3500m
  ↳ I'll use 33 kHz

$$\boxed{\text{Height (Altitude) of Flying} \propto \frac{1}{PRF}}$$

Why ?

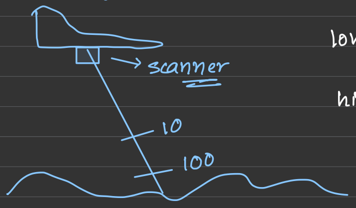
① sequential firing of pulse



② signal strength
Power of laser pulse = constant
higher I am, I need more power, But total power of instrument = constt.

- If I am flying high / flying low and PRF = fixed.
  What will be effect on data density.
  For low flying — more data density
  For high flying — less data density.
- That's why to keep the same data density, I'll have to have high PRF.

- Total Power Consumption of a scanner = constant
- Average Power of scanner = constant

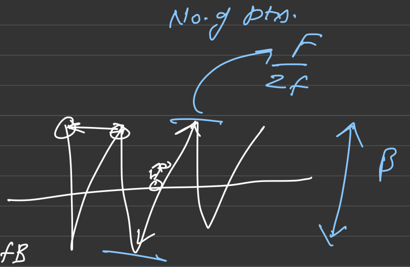


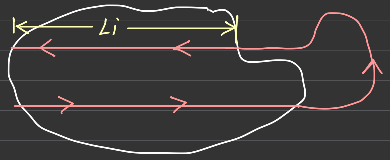
low height — more power
— can ↑es PRF.
high height — less power
— have to ↓ PRF.

Relations ⟶ Approx. relⁿ

$\rho = \frac{F}{BV}$ ⟶ PRF
↙ Bank

- Spacing in along dirⁿ = $D_A = \frac{V}{f}$
- Spacing in transverse dirⁿ = $D_T = \frac{2fB}{F}$
- Total time $\boxed{T = \frac{\Sigma L_i}{V} + \text{time for turn}}$ ⟶ minimize this.
  ↳ cushion period included.



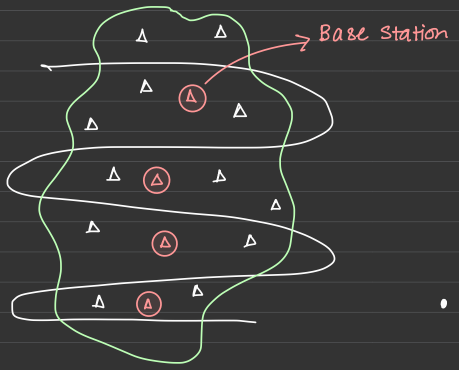


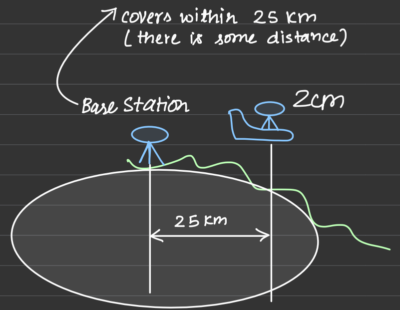
- Using some softwares — one can do flight planning
- A student ⟶ Genetic algorithm based approach.

ρ
a
overlap 10%

φ
H
V
f
θ
PRF
⟶ By varying them which combination gives
T = minimum
(with specification ρ, a, 10% overlap)
↑
specification
(user)

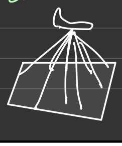
Project Execution steps :-





- Ground Target — to check accuracy.

Lidar Simulation — mathematically generate surface and fly.

 → eqⁿ of plane

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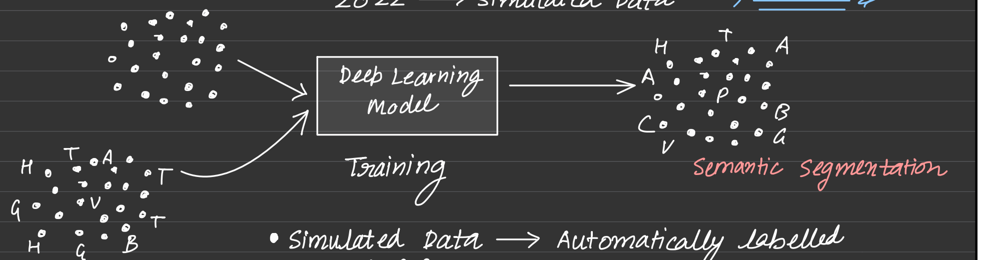
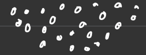
LiDAR Simulation

Why simulator? 2003–2007 → For Education
2022 → simulated Data → BIG THING

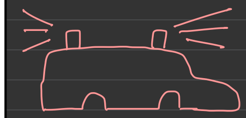


① Label

- Simulated Data → Automatically labelled

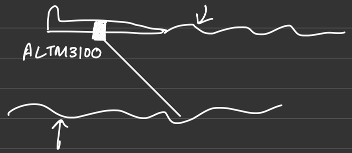


- Actual Data → You've to label !!

 → ① capture the data
② make sense what these objects are.

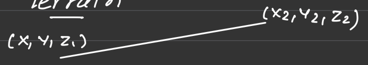
Autonomous vehicle
↳ Training using simulated data → Big thing

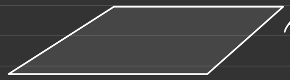
- Tesla – based on camera only. — a vehicle which it was not trained for – it couldn't detect it.



$\Delta t = \frac{1}{500,000}$

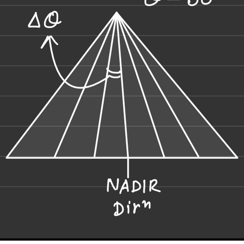
Terrain



 $Ax+By+Cz=0$

$\Delta t =$

500

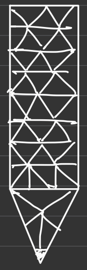


- In 1 sec, total no. of points = PRF

$$\Delta\theta = \frac{\theta}{\left(\frac{PRF}{2f}\right)}$$

How do games generate those terrain?

- They make use of something called mesh.
- They create objects with the help of triangles. They make use of Δs (triangles) and with the help of it they are able to draw the coordinates of the objects that are in the games.



→ It creates a network of Δs (triangles) to make a object

→ Mesh or TIN (Triangulated Irregular Network)

 Texture
↳ computer game
↓
Beautiful terrain

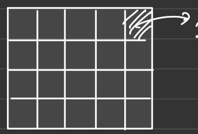
Computer Games
↳ Beautiful Terrains
① Use of Δs (Triangles)
② Texture

Game Engine (Unreal) — Unity, etc → We use the terrain made by game engines.

Raster

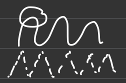
DEM (Digital Elevation Model

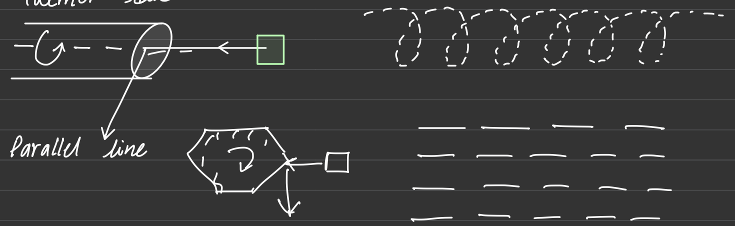
DSM (Digital surface Model)



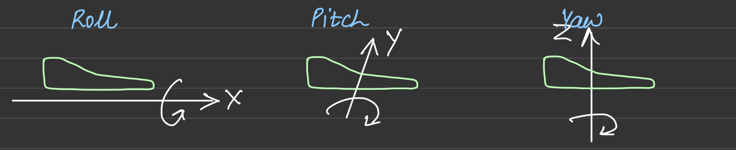
Digital No → Elevation

sinusoidal
zig-zag
Palmer scan





Attitude of the Aircraft

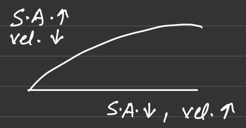


Movement about X axis

Movement about Y axis

Movement about Z-axis

Why aircraft is lifted up ?

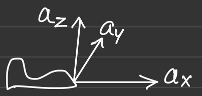


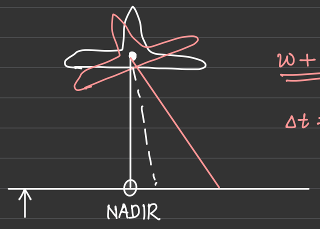
→ This creates the uplift.

- Flat at bottom, but more surface at top.

Trajectory component

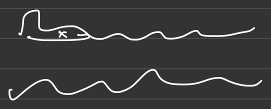
① Aircraft has got attitude
② Acceleration of the aircraft is not constant.





$w + n\Delta\theta$

$\Delta t = \frac{1}{PRF}$



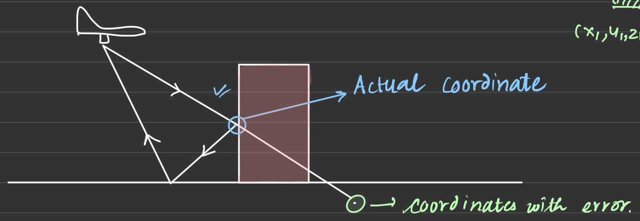
Acceleration Simulation

- Analogous to acceleration actually computed by IMU.

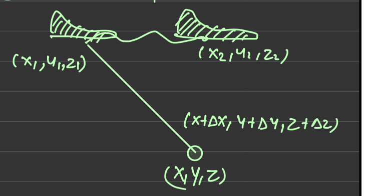
Attitude

ERRORS → Error is always there.

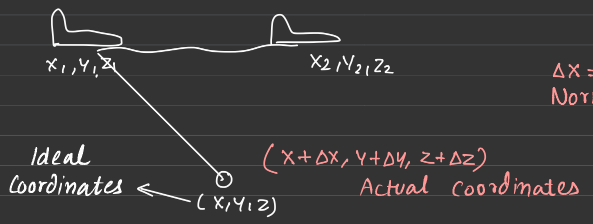
- Multipath



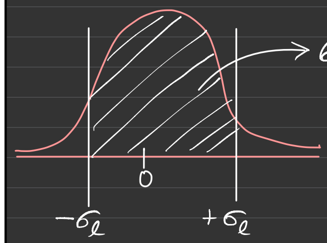
Calculation of Coordinates



INTEGRATION OF COMPONENTS



$\Delta X = N(\mu_x, \sigma_x)$
Normal Distribution



→ 63.8% of the total curve.

$$\boxed{l_a = l_T + \Delta l}$$

→ Randomization

We consider it to be a normal distribution instead of the uniform distribution.
(Randomization — to pickup values of error)

$\Delta X = f(UPS, R, \theta)$ or straight at (X, Y, Z).

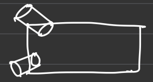
Smiley Error

SMILEY ERROR

- A type of systematic error that can occur in LiDAR data.
- 'smiley error' refers to shape of error which resembles the smiley face in the data.

Uses

- Training
- Generating labelled data for deep learning – Why need.
- Lot of applications for autonomous vehicle training.
- Using Mobile LiDAR Simulation.



$\text{XYZ} + \underline{\text{RGB}} + \underline{\text{L}}$
Color   Label
↗ Deep learning
↓
Tree
Road
Building

Add random noise to Image w/c

Applied Institutions
– CARLA
– Carsim
– Datogen
– Nvidea Doivesim

- Limitator 2.0 → Terrain has mesh → more accurate → working on mesh.

APPLIED INSTITUTIONS :-

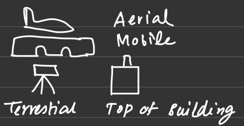
* CARLA : An Open-Source simulator for autonomous driving research.

* CarSIM : A software tool for simulating the dynamic behaviour of passenger vehicles and light-duty trucks.
↳ There is truckSIM, bikeSIM, carSIM, etc as well.

* Datagen.tech : It provides synthetic data for virtual reality, augmented reality, computer vision, and artificial intelligence, namely self driving cars, robotics and IoT security.
↳ Synthetic Image Datasets for Computer Vision.

* NVIDIA DRIVE Sim: An end-to-end simulation platform.

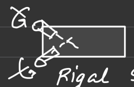
Generic



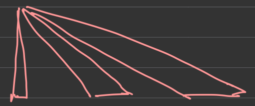
Streetmapper — Mobile Mapping system.

400,000 PRF (pulses/sec)

800m Range



Rigal Sensors



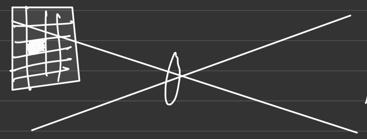
→ As dist ↑, the range ↑

* ESRI 3D City Model

* Simulated Data XYZ RGB Label

↳ It already have

Camera Simulation.



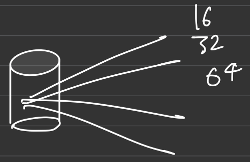
RGB value

Velodyne

Velodyne Sensor

— 100m

— Single Return



LAS File Specification

XYZI RGB L ← Classification

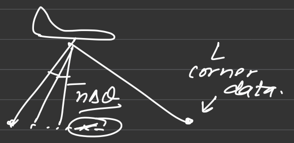
↓

8 dimensional data.

T, L, nsθ, φ, Return

↓

status of overall



ply, pcd — Binary

csv — ASCII

Byte ↑ made of bits

BIL, BSQ

Image RGB

Header

VLR

Data Record

EVLR

? [

4000th

$$\boxed{X = (X' \times x\text{-}scale) + x\text{-}offset}$$

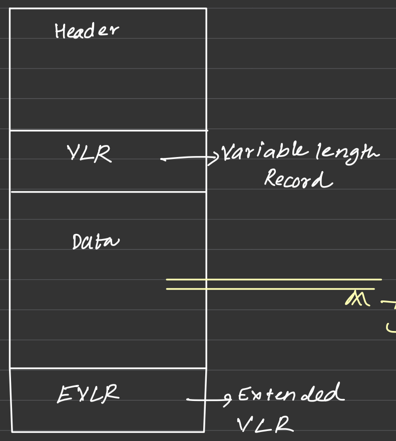
↓ To compute the coordinates

↓ In computer X' is stored and not X.

X,Y - Scale
X,Y - Offset
↓
Convert to any coordinate

Portal for LiDAR Data

↳ Public Fund
Should be Freely available.



1000th Data = ?

{ Binary ASCII
→ Write code for this. } ?

→ 1000th Record
↳ Know where the Return Time is stored.

XYZ I RGB L
θ, T, Return ?

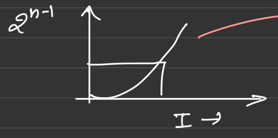
Header → Not textual format → In binary format only in computer stored

↑
657285313
62~~

7285313

0·01

- Intensity → Discritizing in a number.

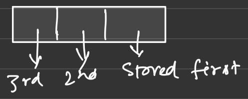


Sensor Dependent
A to D

- Return
- Scan Fly
- Edge → 1 ↳ It is data of edge
  0 → Not edge data.

- RGB
- GPS Time, etc.

LAS ↴
- little-endian format



- Data Types used

- signed
  unsigned → Larger absolute value can be stored.

- code → to check whether LAS file or not
  First 4 bytes = "LASF" → Then LAS.

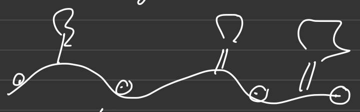
- GUID – some data about project

- Version 1.0
- System Identifier - which sensor generated it. RIEGL, etc.
- Which software?

Point Data Format ID
0
1
2
3

scanAngle Rank → unsigned ~ angle can be -ve also.

Key-point →



↳ some point on ground which are important to create DEM (Digital Elevation Model)

Withheld → Because of some reason, it is not included.

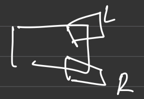
1.4

classes
1 – 64 are defined
65 – . . . are reserved

Extension of Scan Angle Field → More PRF → finer angle resolution
2 bytes ↗

More data

Sensor Channel



Left channel on Right channel.

Lidar Data Generation
Flight Parameter
Data Simulation
} Done ✓

Divergence — Doesn't get fully focused laser beam — divergence is there.

- Diffraction Controlled IFOV. $IFOV_{diff} = 2.44\frac{\lambda}{D}$

$\vartheta \propto \lambda$

$\vartheta \propto \frac{1}{D}$

D ↘ Diameter of aperture (from which it was fired)

- We want divergence to be small. ⇒ $\lambda \downarrow$ or $\underbrace{D\uparrow}_{\downarrow}$
large D not possible

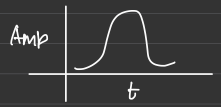
∴ We achieve small divergence by small 'λ'.
That's why Light waves are used.

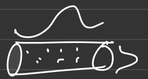
1mm — 1m ✗   400nm — 700nm we use

Ranging

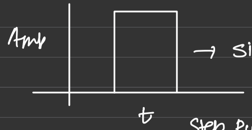
measure dist. b/w two pts.

Two technologies for ranging → ① Pulse — light on & off
② Phase — light is continuously on





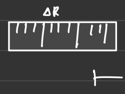
* For Gaussian Pulse
↓
How to measure Time of



→ simplified

Step Pulse

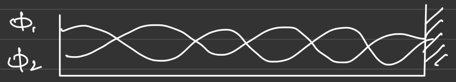
$R = \frac{t_L}{2} \times c$



Ruler

Range Resolution :- $\Delta R = \frac{c}{2} \Delta t_L$

CW Laser



x_1 x_2 x_3 x_4

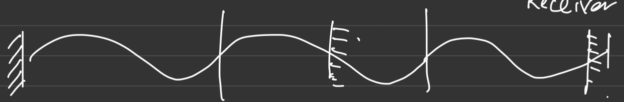
$\overline{x} = \frac{\Sigma x_i}{n}$ most probable value

↓

more nearer to any of x_is.

$\Delta\phi = \phi_2 - \Phi_1$

1 Hz
10 Hz
10KHz



$$\boxed{t_L = \frac{\Delta\phi}{2\pi} T + \underbrace{nT}_{}}$$ → no. of wavelengths.

$T = \frac{1}{f}$

$\Delta\phi =$

$\frac{t_L}{2}$ → for Range

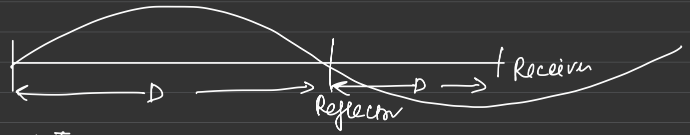
$D = \frac{M\lambda}{2} + \frac{\Delta\phi}{4\pi}\lambda$ } → The same formula →

→ No. of wavelengths filling the space.

M – unknown to us?

- How to measure M? Decade Modulation → Look later

$t_L = \frac{\Phi}{2\pi} T + nT$



$R = \frac{c \times T_L}{2}$

$\Delta R = \frac{1}{4\pi} \frac{c}{f} \Delta\phi$ – Resolution of phase diff

→ freq. of wave employing

smaller the wavelength, better the resolution.

Decade Modulation

Sending multiple wavelengths, we can find 'm'.

M = ?

Principle :-

L.S. 900 nm → carrier wave
9000 nm
90,000 nm

Other signals are carried on top of this.
↓
use of modulation

1
10
100
1000
10008

} In actual life we do by

3.(25)

↓

more preise part from shorten wavelength

What does Modulation mean?

ToF → Pulse

$\Delta R = \frac{1}{2} c \Delta t_L$

↓ controlled by precision of time

$\Delta R = \frac{1}{4\pi} \frac{c}{f} \Delta \phi$

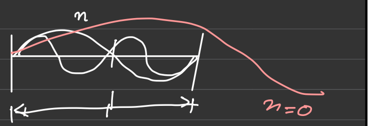
↙ freq. of wave on

$\underline{\underline{\Delta R \propto \lambda}}$

↓ how precisely I am measuring the

$R_{max} = \frac{1}{2} c \, t_L max$

↓ Also depend on Intensity and reflectance of the wave.



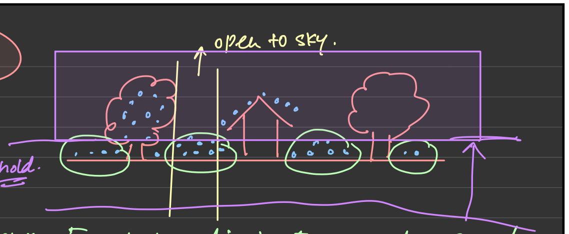
max. range that can be determined.

max. range depends on phase diff.

$\boxed{\text{Max. Range} = \frac{\lambda}{2}}$

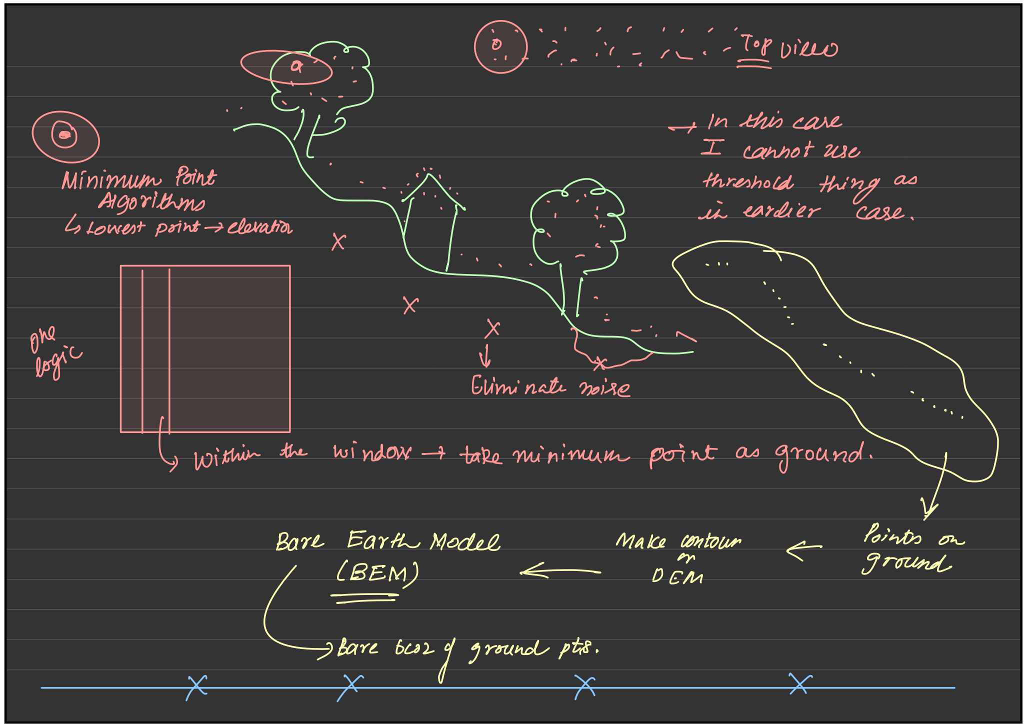
Lab

Ground Filtering



contours

↳ To make contours I need to eliminate the non-ground points. but they are in X,Y,Z I.



Minimum Point Algorithms

↳ Lowest point → elevation

One logic

↳ Within the window → take minimum point as ground.

↓ Eliminate noise

→ In this case I cannot use threshold thing as in earlier case.

Points on ground → Make contour on DEM → Bare Earth Model (BEM)

↳ Bare bcoz of ground pts.

Range Accuracy

CW – short range → $\bar{R}_{cw} \propto \lambda_{short}$

Pulse – large range that's why we use it in

σ. $\propto \Delta t$

$\propto \Delta \phi$

$\lambda = 1\,mm$
$\lambda = 100\,mm$ } → Resolution same.

By using shorter wavelength
CW - more accurate than the pulse

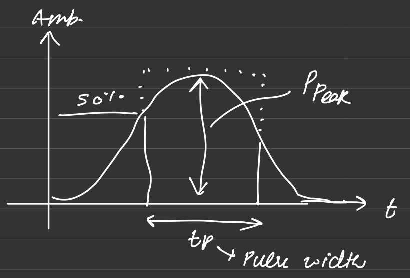
Pulse → accuracy depends on time measurement
↳ less accurate.

CW → μm accuracy
For short range more accurate

Pulse → can't achieve μm accuracy.
Limited accuracy
↓
depends on time measurement accuracy.

Ranging Principles < Pulse / CW

- Resolution depends on time
- phase diff., shortest wavelength
- Accuracy more than resolution



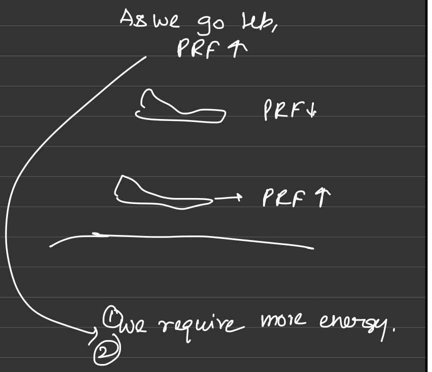
Peak Power × t = Energy.

Energy in one pulse = Ppeak × tp

☆ Imp. $P_{av} = P_{peak}\, t_p\, F$

If P_{av}, t_p ~ constant.

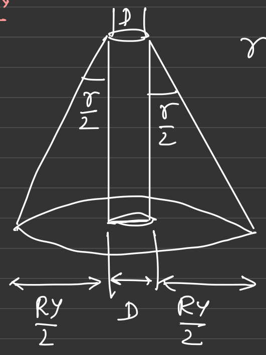
$\boxed{P_{peak} \propto \frac{1}{F}}$ → PRF



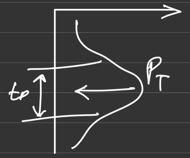
① we require more energy.
②

V. Imp. - slide 3

LASER EQUATION



γ → total divergence



Area of footprint
$A_I = \frac{\pi}{4}(D + R\gamma)^2$

Power Density (Irradiance Wm^{-2})

$\phi = \frac{P}{A}$ (M)

M - transitivity coeff. of atmosphere

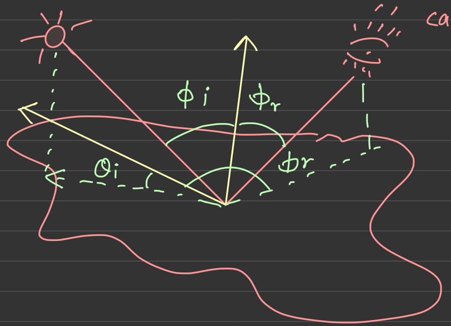
Total Power impinging on this

= Power Density × Area of target

=

Considering target as ↓ Lambertian DRPF target

BRDF (Bidirectional Reflectance Distribution Function)



Nothing but Spectral Reflectance ↓

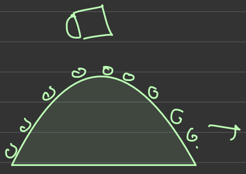
$f_\lambda(\theta_i, \phi_i, \theta_r, \phi_r)$

$= \frac{E_\lambda(\theta_i, \phi_i)}{L_\lambda(\theta_r, \phi_r)}$

λ for corresponding to a wavelength.

↓

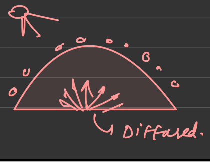
Reflectance



→ Different Reflectance

BRDF

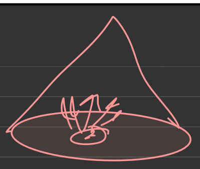
Lambertian →



Irrespective of source of light, the reflectance is same in all the direction.

Radiant Intensity = $\frac{BRDF}{\pi} = \frac{\rho\, \Phi_{tar} A_{tar}}{\pi}$

every unit solid angle

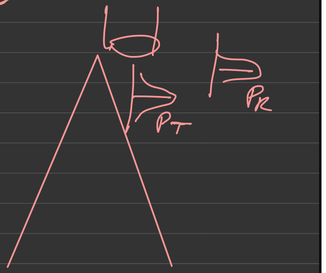


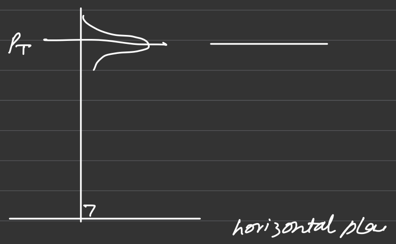
Laser Eqn

$$\boxed{P_r = \rho \frac{M^2 D_r^2 D_{tar}^2}{4R^2(R\gamma + D)^2} P_T}$$

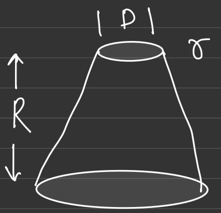
① P_T —
② M — transmitivity of atmosphere
M = 0 to 0.1

③ Dia. of target.
④ Dia " receiver
⑤ γ → also.
⑥ R, D → Inverse





SOLDIER ANATOMY



EYE SAFETY

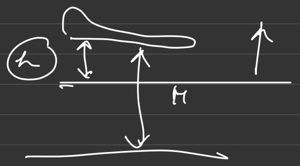
700 -	5.10^{-7}
1050 -	5.10^{-2}
> 1400	100 -

Peak Power means beyond this value my eye will get damage.

Smaller peak value → damages more.
large peak value → damages less
↓
more safe.

Eye Safety also depends on Range.

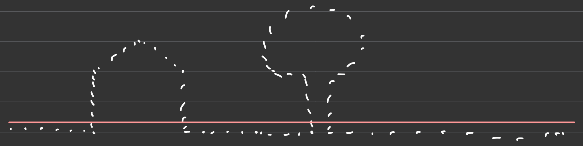
upto is the laser is not safe.



Range also upto which is not safe, we should stay beyond that range.

DSM → BEM | DEM

Threshold Algorithm → For no undulation

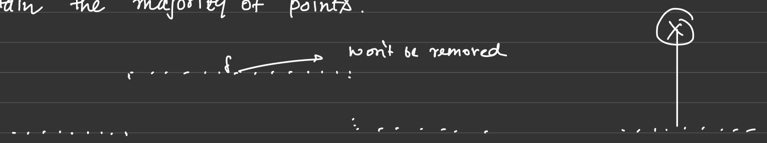


Retain all points for which
$z <$ Threshold

DEM

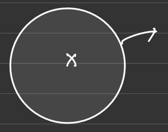
Majority Filter

Retain the majority of points.



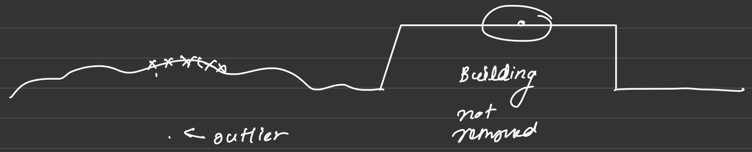
Not a robust algorithm (has limitations).

Local Minimum → Raster
→ TIN



If the dist. b/w two pts is less than threshold

Not robust.



Local Median

slope b/w pts. neighbour.

Ensemble → Algorithm in a boolean operator.
↪ Not very imp.

CSF Algorithm in CloudCompare

↳ Generate the Ground class and then observe it.
But still many errors.
Errors can be false positive.
" " " false negative.

Deep Learning → Very good work
1000. of data of India.

Deep Learning Algo.
Ground class → generate
↪ Ground → Edit → Final Ground.

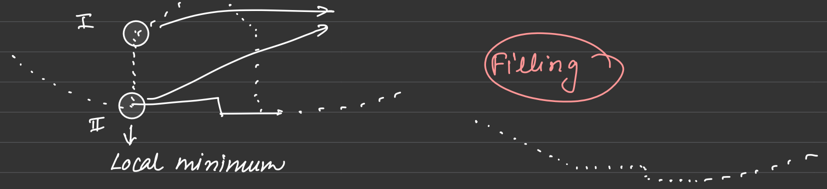

Raw Lidar Data → Algo. → Ground Erroneous → Manual → Ground corrected

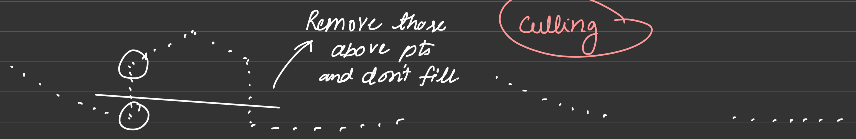
Use this both data to train a deep learning model.
↓
Model will be trained as a person is trained.
↓
Then from erroneous ground the corrected ground can be.

Possibility from DL ☆

Culling and Filling

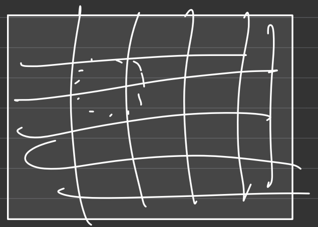
Replace the elevation of pt I to elev. of p II



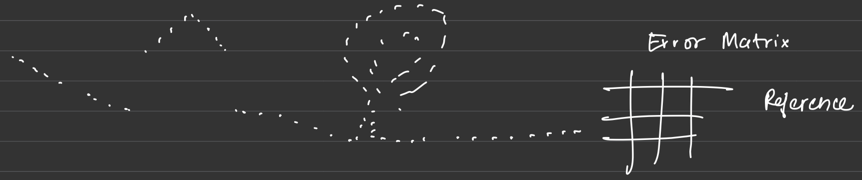


Remove those above pts and don't fill

Interpolation of bare earth model



Validation of BEM



Binary Classification
- ground pts.
- unground pts.

	G	AG
G	n'	m-m'
AG	n-n'	m'

Error Matrix or Confusion Matrix.

✓G
AG
AG
⋮

G AG
n m

$n+m=N$

$n \rightarrow n'+m-m'$

	G	AG
G	n'	m-m'
AG	n-n'	m'

False Positive

User Accuracy

False Negative

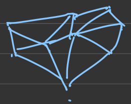
Producer's Accuracy or Recall

$\frac{(n'+m')}{(n+m)}$ → overall Accuracy!!

Precision:- true

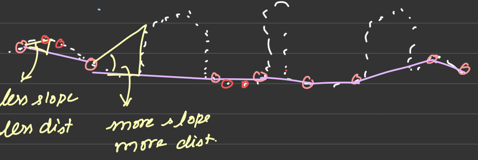
Generate quality of work!!

Terrasolid → Algorithm for Alexandar

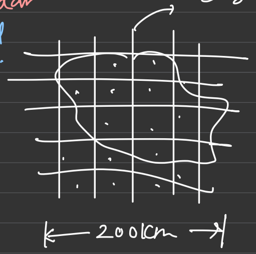


Triangulation.

Identify ground pts definite.



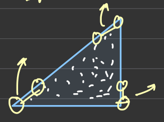
Iteration Angle
Iteration Distance.



Large grids.
↓
minimum ground pts.
↓
Lowest pt. is ground.

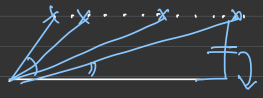
① Points definitely on ground.
② Triangulation.
③ Judging of a potential pt. is on ground or not.

3pts.

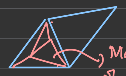


If dist. if more than threshold, we'll call it AG (Above ground)

Why we dist. if only angle is sufficient?



→ 8°
→ 3°



Make other set of triangles.
Triangles attaching to ground → size of Δ becomes smaller.

Bush | stone.

Distance & Angle. still be small.
↓
Now we need to reduce threshold.

Limitation:-
* Highly Parameter dependent

Terrain Angle

TIN slope
triangle slope
individual.

Similar → CSF Algorithm.

OSM – Open Series Map

- Building are just represented as blocks even if it has got some roof.

When will sun-shine reach a particular building?

We want to predict how much sunshine will reach a particular building. For how long sunshine remain. For that we need the data of sunshine throughout the year. We have to accumulate it.

Today at 9AM, sunshine is reaching a building. Will it reach the same building at same place at 9 AM? No!! → Why?

The inclination of the sun changes throughout the year. → Important

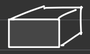
In summer – sun may be further up – sunshine may not come there.

→ We can use this information to place a solar panel if we want to do rooftop solar panel installation.

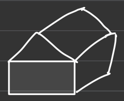
→ Govt. want to derive – 40% energy – from solar panel.

Level of Detailing

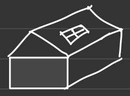
LoD0 →



LoD1 →



LoD2 →

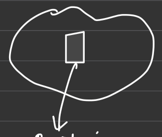


LoD3 →

LoD4 → Virtually you can walk → inside details are also present.

Where are buildings in the point cloud?

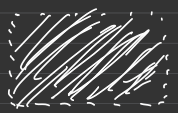
Our mind has "domain knowledge"



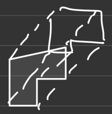
Boat in sea

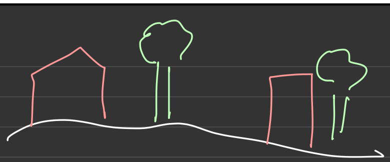
car on road

Based on detection of planes



- confusion in trees building bus, etc
- In LiDAR data, geometry is the criteria only.
- Aerial images → RGB XYZ } Deep learning Model.
- Ground Plan — floor plans are already mapped

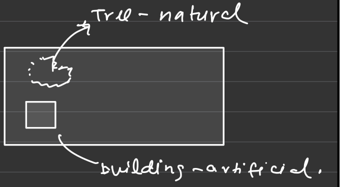




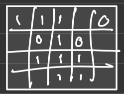
DEM

BEM

nDSM



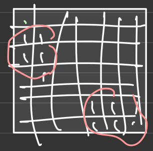
Applying threshold on DSM — below
↘ above

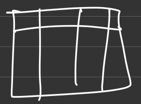


<u>Blobs</u> ⟶ uniform areas trying to put together.

A(i,j)
R(i,j)

Forming blobs of 1.

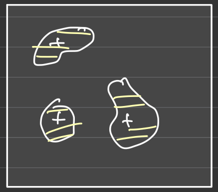




= 1
= 2
= 3
= 4

<u>Concept</u>.

- size not changing — building
- size changing — non-building blob
- location and size of building not change.



$\Delta size = (size_i - size_{i+1}) / size_i$

Approach: Classification.

nDSM — One band. ④

{ LiDAR Data
Photograph

Intensity ⑤ Image
↳ ortho

RGB → ortho photo
↓
same ortho projection as LiDAR data.

①
②
③

↓
RGB. orthophoto.

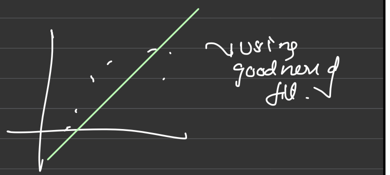
* size of nDSM, Intensity Image
↓
Same ☆

- LiDAR - ortho
- camera image - perspective ?

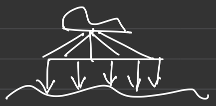
↓
Why ?
↓
Exam ☆

- Intensity image — ortho

Approach: Using Planar surfaces.



LiDAR Data is Ortho



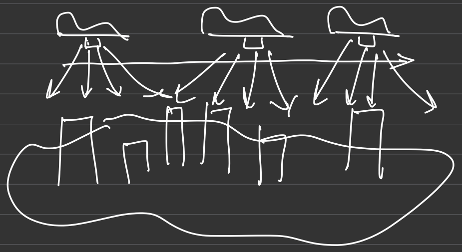
- LiDAR Data ⟶ Orthographic Projection
- Orthographic Projection is where parallel lines in 3D world are projected as parallel lines on 2D surface. This represents the relative positions and sizes of objects in the scene without any distortion due to perspective.
- LiDAR Data → not inherently orthographic but it can be represented in an orthographic projection.
- LiDAR Data is orthographic meaning it represents objects in a flat, two dimensional manner,

Camera image is perspective

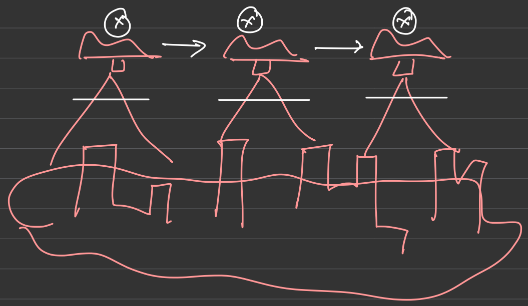
LiDAR Data → Orthographic → Represents objects in a flat, 2D.

Camera Images → Perspective → Represents objects with a sense of depth and 3D.
↳ Relief Displacement.

Camera → Captures depth and 3-Dimensionality
↳ Relief Displacement.



⇒ LiDAR Data
↓
Orthographic.



→ Camera Image
↓
Perspective

- Make a pair of two near points (in geography)
- Get a m and c value

Hough Transform.

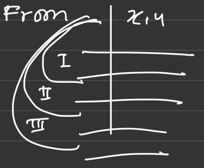
- Trying to fit a line in a data.

(x_1, y_1)
$(x_2 y_2)$

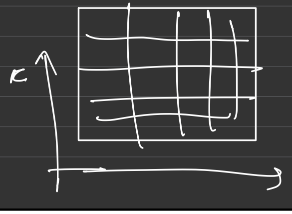
m, c

Problem

From

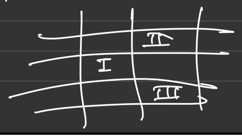


→ A line having more than 10 pts.

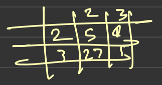


called: Hough Room → Discritized Parameter Space.

Discritize this space → make grids.

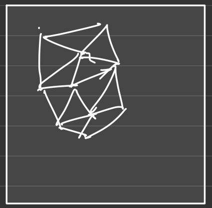


Hough Room — Populated — frequency Plot. —shows how many such lines.



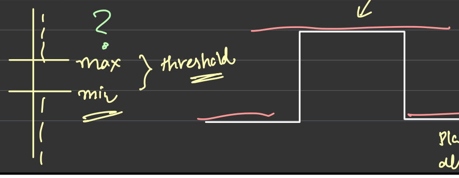
20 < line → keep it → Remove other.
↓
- length of line
- & data density

Hough Transform — 3D — We want to fit a plane there.

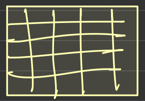


Project LiDAR Data — 2D —

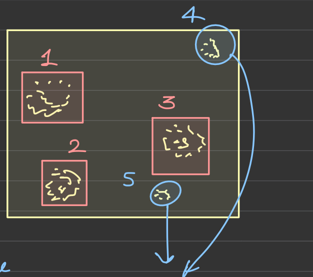
TIN



- Rasterize the points.
- Use Image Processing algorithm for blob identification



- Give Different ID to blobs
- Map blob back to LiDAR Point.



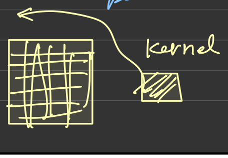
Morphological Closing

- without ↑sing size of overall
- Fill the cells.

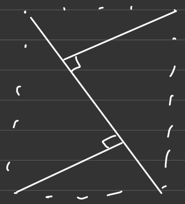
- Fill the holes.

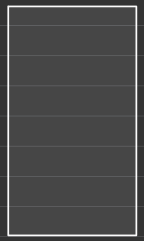
Remove smaller < threshold points

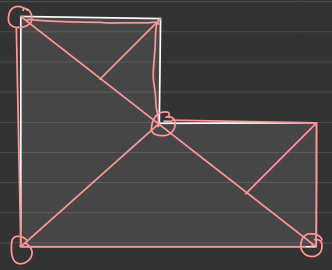
Edge Identification



Line Fitting to Edge Data Points.







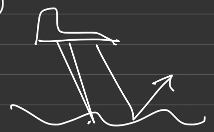
Contour Triangulation Algorithm ✱

- Buildings are generally orthogonal

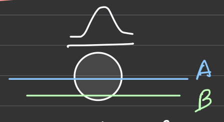
☆ Paper — GeoCarto International.
Rajneesh Singh, Prof. B. Lohani.

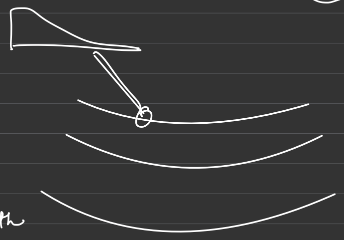
Building Identification ⟶ Entire methodology.

- Type of target reflectivity (diffuse, specular)
- Laser sensor / sensor detectivity

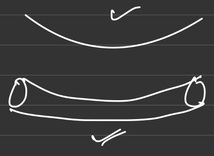


Minimum objects





A has more than B →
- Bcoz of more length
- Max. Intensity is there



→ Reflectivity is higher ✓

wood → Reflectivity is poor ✓

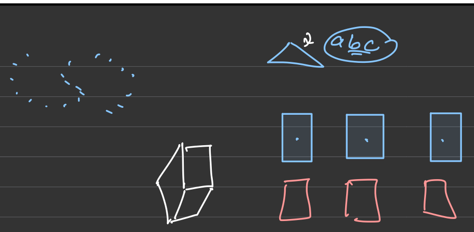
↳ Even though it is big!

but reflectivity is poor

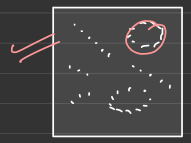
∴ no coordinates detected.

Laser Scanning Pattern.

Planar surface



* Whether some points in dataset follow a model or not ?
What is the eqn of that model



using edge detection
(canny edge detector)

fit in a circle.

eqn of circle

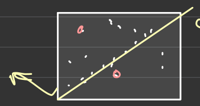
RANSAC (Random sample concensus)

If there is any line?
If yes, what is the eqn.

inliners – those datapoints that fit in the model and lie inside the model.

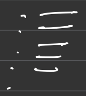
outliers –

· Start with a random sample for a chosen model



↳ consensus set

X Y



2 – select any two points.

consensus set – those pts which are inliners.

threshold – length of line. .

Iterative Method.

$m=2$.

Prob. of inlier $= p^m$

Prob of not choosing inlier $= (1-p^m)^K \rightarrow$ Prob. of not choosing any inlier.

Pdro $= 1-(1-p^m)^K$

Prob. of getting a model / consensus set $= 1-(1-p^m)^K$

No effect of n.

p – estimated
m – from model we fit , For lines $m=2$
For planes $m=3$.

Iteration should be more than

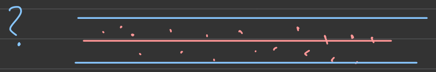
---

x, y, z

$m=3$.

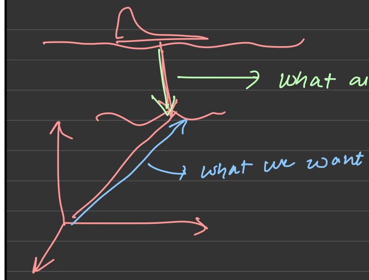
How to decide the distance threshold for LiDAR Data

Error

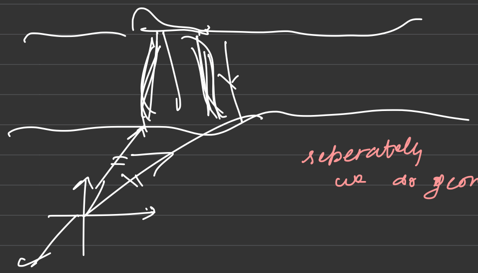


precision of sensor
± dist.

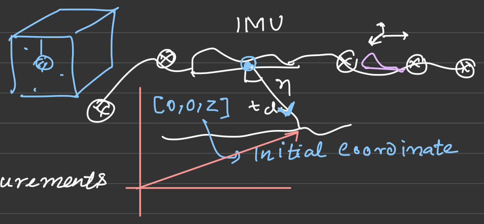
data threshold
↳ minimum size of rooftop
&
data density



How to do that?



seperately we do georeferencing.



conservation of momentum

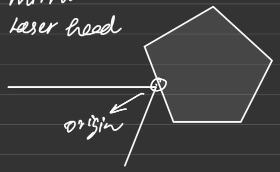
Primary Measurements
① d(t)
② η
③ XYZ
④ Roll, Pitch, Yaw

Each pt. is geolocated.

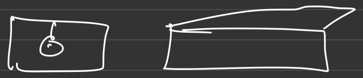
Geo-location : Meaning? → Paper

Prof. Salil Goel's student
↳ Backpack based laser scanner
- has IMU, GPS, scanner
- in a bag.

Mirror
laser head



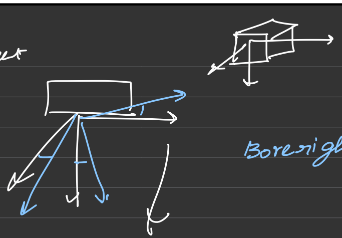
IMU Reference System (Body) — It gives roll, pitch, yaw.



Angular Rates.

CIO → True North

IMU & Instrument



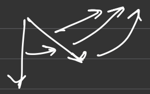
Boresight values
$(\alpha_0, \beta_0, \gamma_0)$

- Orientation is diff.
- Translation also b/w them

Misalignment

bcoz anything is not perfectly parallel.

Imp: ★ Instantaneous Axis System — Time varying coordinate system ~ Red line.

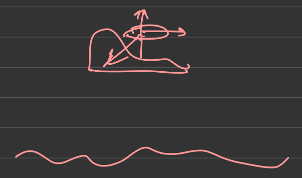


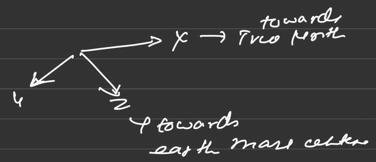
[0,0,z) [0,0,z']

Gyros

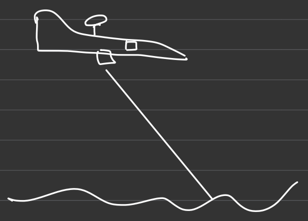
Earth Tangential Ref. System.

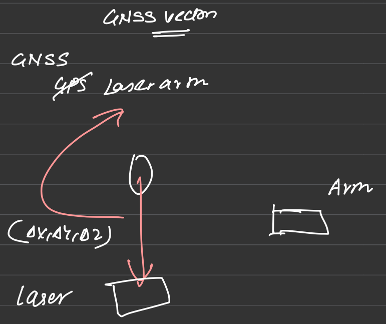
↳ At GNSS Antenna





Rel^n b/w all c.s.s





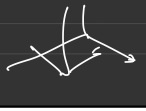
IMU — (Roll, Pitch Yaw) → ET Ref. System

(ω, ϕ, κ)

WGS84 - ECEF
- coordinate system.
- RH c.s.

GNSS observation done in this.

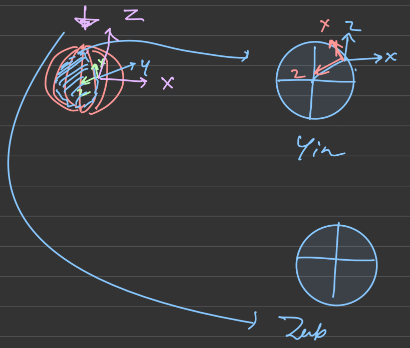
Cartesian
WGS
ECEF



---

conversion → How? — go back — Known to us.

$(\lambda, \phi, h) \longleftrightarrow (X, Y, Z)$

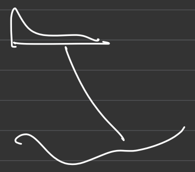


$R_y\left(\frac{\pi}{2}+\phi\right)$

↳ positive Rotation

Clockwise −ve
Anti " +ve

Entire Georeferencing Process



$[0,0,z]^T$

↳ Rotate it to bring it to black c.s. (laser)

$R_x(\gamma)$ — scan angle

↓

Now bring it to blue c.s. sitting at laser head and ∥ to IMU.

$R_x(\alpha_0)\ R_y(\beta_0)\ R_z(\gamma_0)$ — boresight values.

↙

Now GNSS vector

$T(\Delta A_x, \Delta A_y, \Delta A_z)$

↓

Now bring it to ET ref system from IMU c.s.

$R_x(\alpha)\ R_y(\beta)\ R_y(\omega)$ Roll Pitch Yaw

↙

sitting at antenna

WGS 84 coordinate system

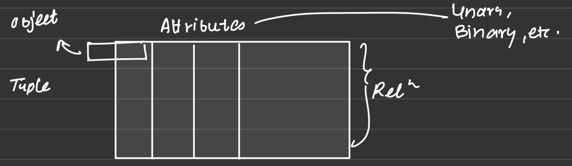
$R_y(\pi/2+\phi)$
$R_z(-\lambda)$

↑

$T(x, y, z)$
translate to bring it at centre of earth.

$[0,0,z]^T$

- RDBMS → Relational. → We are creating relations.
- Entity or Object
- Normalisation → Generation of these relations.

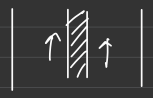


- Used in GIS
- Basic Queries using SQL.
- Joining – Inverse of Normalisation.
- SWAMITVA Project by Government
  Parcels → Village Area

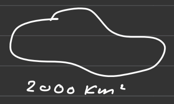
UPID
UPIN

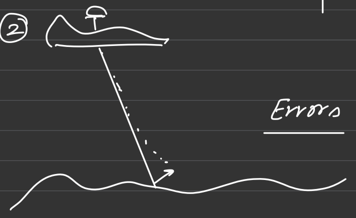
L-14

① Process of checking?



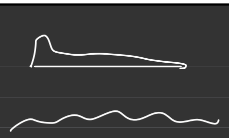
-d
- accuracy
⋮



②


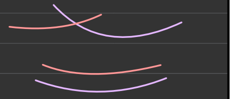
Errors

Point where it hits,
the point cloud will be something else.



Aerial LiDAR – Data Quality Check Parameters.

Why overlap.
① Neck to Neck
↓
Data loss
② Quality Check.

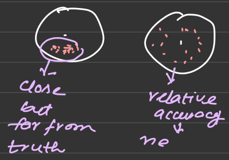


- Vertical Accuracy
- Horizontal Accuracy

] → Absolute
↓
When comparing it with actual ground and ground system.

- Relative Accuracy

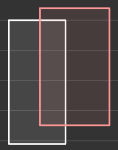
] → talks about precision.
↓
nearest to each other.



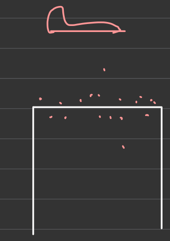
- Data voids
- NPS (Nominal Pulse Specific) and data points.
- Spatial distribution (clustering and uniformity)

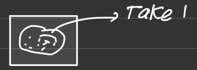
LiDAR Overlap.

Code → to check overlap during flight.



Rel. vertical accuracy





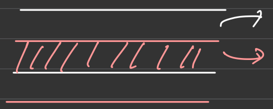
- Fit a least square plane.
- Take ⊥ dist.
- Calc. deviation.

σ_z

- $$\sqrt{\frac{(\bar{z}-z_i^2)^2}{n-1}}$$?

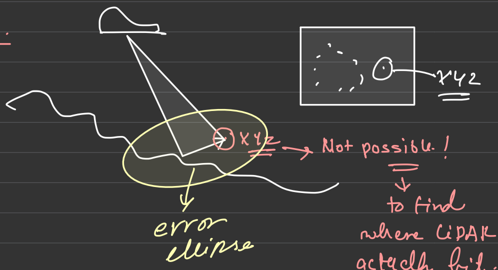
- std. dev → 1σ ⇒ 68.3%
  $\pm$
- std. dev. → 1.96σ ⇒ 95%
  $\pm$

↘ Generally talk abt. 95% accuracy.



Ideally both should be same.

Absolute vertical accuracy.

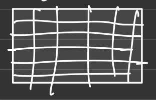


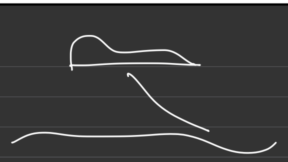
→ Not possible !
↓
to find where LiDAR actually hit.
↓
random errors.

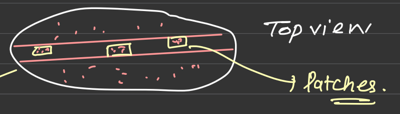
Hypothetical → photonic material.

- Satellite – Complete capture.
- LiDAR – Discrete capture. — Incomplete capture

↳ Using Intensity image → created by interpolation.
↓
Interpolation error
↓
biased !!

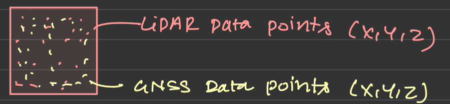


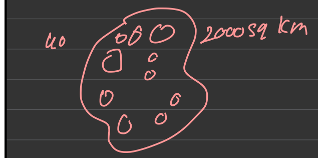




- NVA-Non Vegetated Accuracy.
- VA – Vegetated Accuracy.

Extract Lidar points of those patches





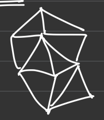
- over different class (like concrete, etc)
- over vegetated / non vegetated.

Thumb Rule:– Accuracy of check points > 3 times altimetric accuracy.

Rem: ⁎ ↳ For any data, accuracy of checking device > 3 times reference device.

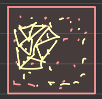
· Say LiDAR accuracy 15cm, ⟶ GNSS survey should be at 5cm. accuracy.

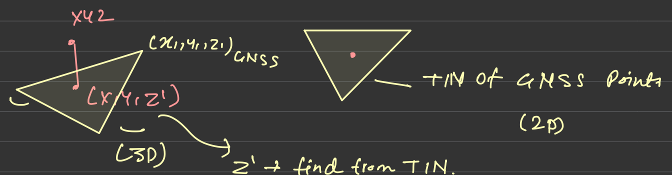
TIN



Δ	vertices	neighbour Δs
1	– – –	– – – –
2	– – –	– – –
3	– – –	– – – –
4	– – –	– –

LiDAR Data & TIN Data not co-located.





z' → find from TIN.

Ideally, if no error, $z' = z$

$1{\cdot}96\sigma \rightarrow 95\%$

① Standard Deviation (R.A.) 1σ, 2σ, 3σ

↳ closeness to each other ⟶ No need of reference data → $Z_i^{ref} = Z_{mean}$

② RMSE (A.A.)

↳ close to some reference data

↳ need reference data.

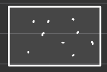
$n-r$

that's why.

vertical accuracy reporting

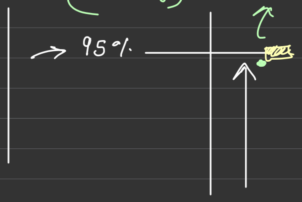
↳ single — first return (only)

VVA (Vegetated Vert. Acc.)



Zi

Zref.



(Zi - Zref.) ← Differences

Look at below 95% data.

Why? Because deviations are not normally distributed.

LiDAR Coordinate Computation

$(0,0,z)\cdot R_x(\eta)\cdot R_x(\alpha_0) R_y(\beta_0) R_z(\gamma_0)\cdot T(\Delta x,\Delta y,\Delta z)\cdot R_x(\omega) R_y(\phi) R_z(\kappa)\cdot R_y(\frac{\pi}{2}+\phi)\cdot R_z(-\lambda)\cdot T(a_x,a_y,a_z)$

Time varying C.S.

Error in IMU & Laser C.S.

Roll Pitch Yaw

Coordinate of GPS Antenna

Laser Instrument C.S.

// to IMU C.S.

Translate to GPS C.S.

Now in Earth tangential system

Now parallel to WGS84 sitting at GPS Antenna of Laser

Now C.S. to WGS84 sitting at the center of earth.

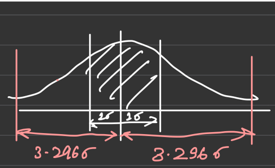
- Translation is usually added but in the case of homogenous system, it is multiplied.
- $[X] = [X, Y, Z, 1]^T$ ↓ X, Y = 0

- GNSS and IMU are in same C.S.

Vertical Accuracy Reporting

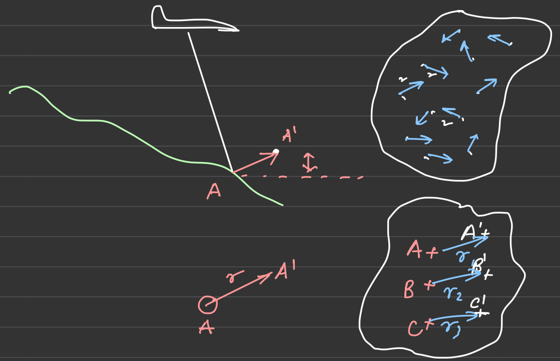
- Patches — for every 50 sq. km, one patch should be there.

1σ — 63% points
1.96σ — 95% points
3.296σ — 99.9% points



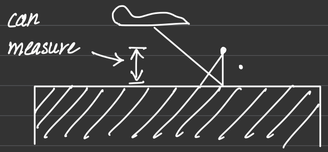
Planimetric Accuracy

- $RMSE_r = \sqrt{\left(\frac{\sum_{i=1}^{n} (r_i)^2}{n}\right)}$
- In case of 2D gaussian Accuracy at 95% = 1.7308σ confidence level



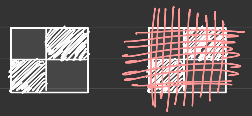
MUST UNDERSTAND :-

- In vertical, easy to compute the vertical component of the point deviation.



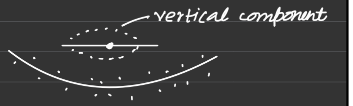
- In horizontal — difficult to find the horizontal accuracy

NO DIRECT METHOD
↓
INDIRECT METHODS → No standard method
↳ If you give one method, people will recognise it.



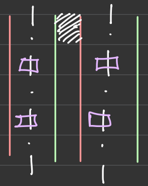
Interpolating → To make it a raster

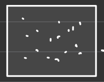
Transmission Lines Approach — Not suceed



NPS (NOMINAL PULSE SPACING)
AND
DATA DENSITY
→ Both are dual of each other (related)

Data Density ≡ PPSM
Points per square meter



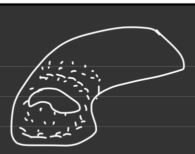


→ TIN

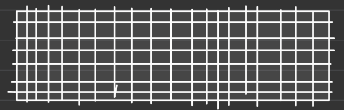
(First return only)

Data Voids — Points without data

- Area > 4 * $(NPs)^2$

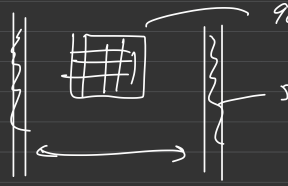


Spatial Distribution of Data



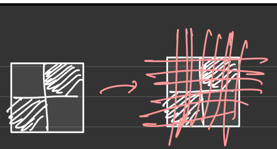
Not taken this area
↓
uncovered.

size of grid = 2* NPS → 95% points should be filled.



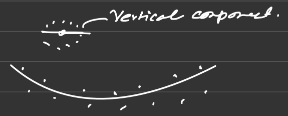
Thumb Rule

Accuracy of data = 15cm
Best we can do C.I. = 3* Accuracy



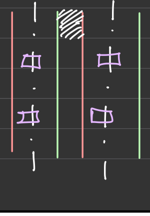
→ Interpolating to make it a raster.

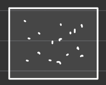
Other Approach :— Not suceed. → Transmission Lines.



Data Density

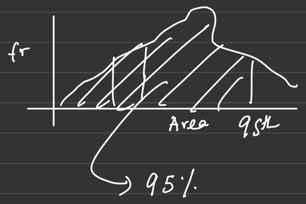
NPS (Nominal Pulse Spacing)
and
Data Density
Both are dual (related) of each other.





→ TIN

first return only



Data Density = PPSM
= point per square m

LiDAR General Specifications

USGS – Specifications

- Uniformity
- Data Density / NPS
-

Interested in

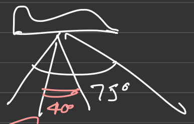
· Atmosphere

· wind speed < 20 knots :– → Topography.

Data Requirement

· Accuracy at 95% CI → VVA
NVA

· overlab → 10% or more



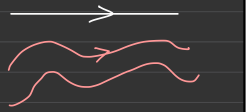
In general

BUT for DEM,
FOV = 40°

As we move larger towards end, footprint size ↑es and uncertainty ↑es.

- SPECIFICATIONS ~ Mainly for DEM. → REM .

Flying Method :– → Follow straight
→ Follow terrain (altitude



· Non-consecutive Turning and consecutive.

Imp: Location of Base Stations

An idiot with a plan

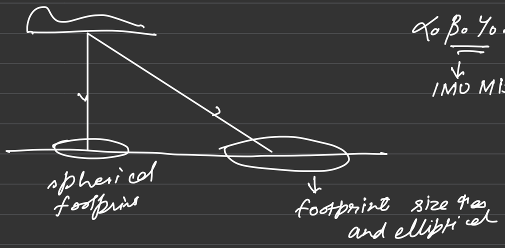
Max coverage of base station : area of 30 km radius.

- If flight lines goes straight more than 30 min. — then IMU goes to sleep., → will not give values.

Aerial LiDAR: Error and Sources

Sources of Error

ⓣ ⓒ ⓓ



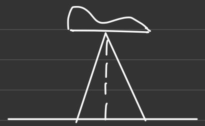
$\alpha_0 \beta_0 \gamma_0$.
↓
IMU Misalignment

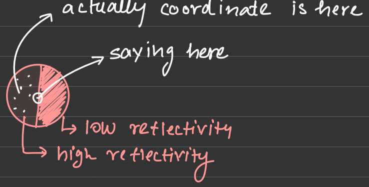
- ω, ϕ, κ — IMU—Instrument < systematic errors / random errors

---

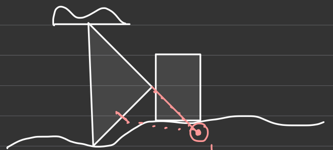
- Vertical Accuracy (Vert.) 3-15cm (1σ) → height is different
- Horizontal Accuracy $= \frac{1}{2000} *$ Altitude $=$ 3-30cm (1σ)
- Horizontal Accuracy is poorer than vertical accuracy
- INS — IMU

- Surfaces can have high reflectivity or low reflectivity.
- Footprint size



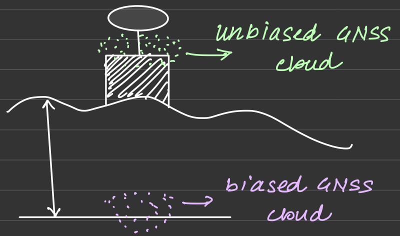


- Multipath Effect



Error in computed coordinate

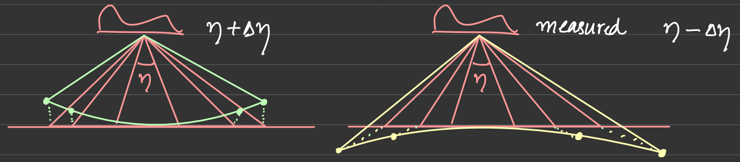
GNSS Bias



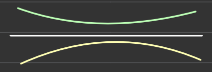
Error in Laser Sensor

ERROR IN SCAN ANGLE

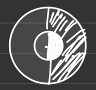
η $\boxed{\eta+\Delta\eta}$ → Measured

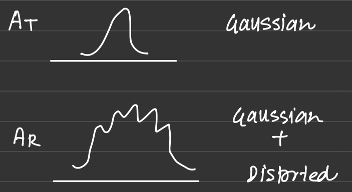


- Flat surface looks like curved in case of error in scan angle measurement.
- Because of bias in scan angle measurement, the nature of surface changes

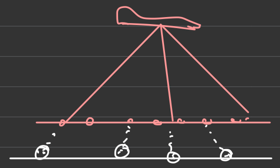


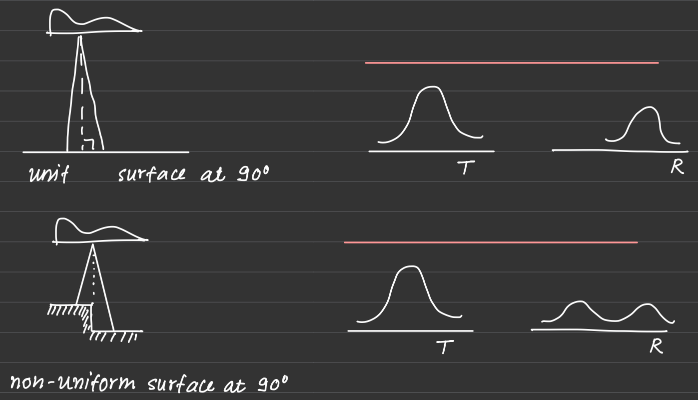
- Larger footprint → more uncertainty

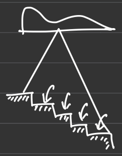




BIAS IN TIME MEASUREMENT
$(t+\Delta t)$







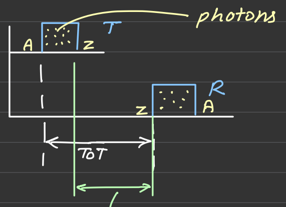
- for low reflectance
  ↓
  high ToT

Shape of return pulse can be different depending on the geometry and reflectance of footprint.

METHODS TO MEASURE ToT

→ How to measure ToT? What methods to use?

- In case of step pulse



Actual ToT is this (∵ photon Z is there bcoz it hits the target first)

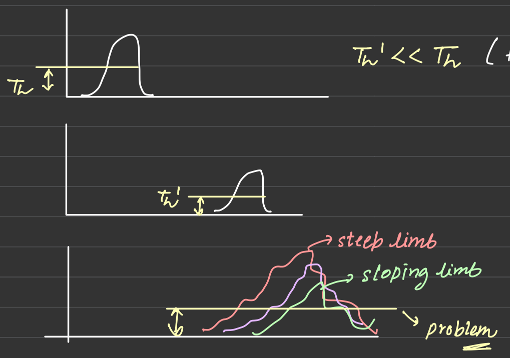
METHOD-1 : CENTROID OF THE AREA OF PULSE

METHOD-2 : THRESHOLD CROSSING METHOD / CONSTANT FRACTION

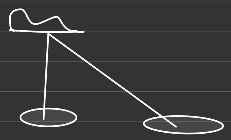
when pulse crosses a certain threshold value.

arrival of returns — when crossing a given threshold

BUT IT HAS GOT A PROBLEM! → RANGE WALK ERROR



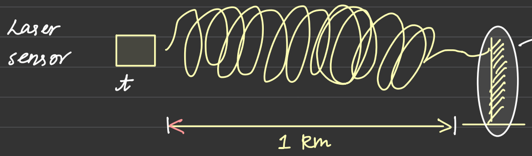
$T_h' << T_h$ (threshold value of return is less bcoz the peak power is less)



Time of return measurement is diff. bcoz of change in material (spectral) reflectance

---

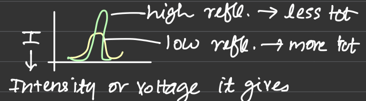
How to handle this problem? METHOD OF CALIBRATION → In Lab



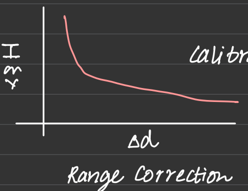
This material will be changing reflectivity from high to low material.

$\Delta d = d - d_m$

reflectivity ↙ actual ↳ measured



Intensity or voltage it gives

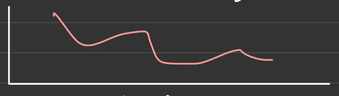


Range Correction

Calibration curve → (Plotted and fitting a model (curve))

$I, V \rightarrow \Delta d$

Got a value of range correction as per Intensity

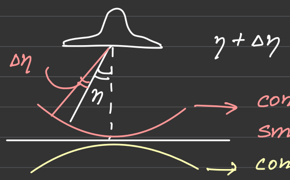


Amplitude of waveform peak

- All instruments have got a calibration curve.

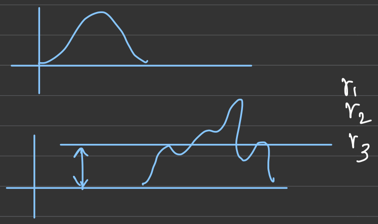
SCAN ANGLE ERROR

- Error — Systematic / Random



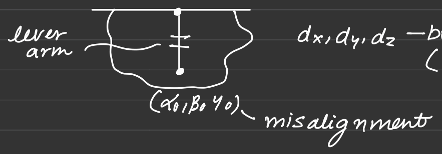
→ convex error or smiley error

→ concave error or sad face error



Error due to sensor position

- Instrument have got errors in position of GNSS, IMU, Interpolation and position of one sensor w.r.t. other.
- IMU Error — Roll, Pitch, Yaw
- What is the role of roll, pitch, yaw given by IMU?
  To establish earth tangential coordinate system below earth surface



d_x, d_y, d_z — boresight values (IMU & GNSS difference)

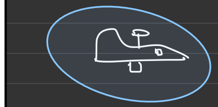
$(\alpha_0, \beta_0, \gamma_0)$ — misalignment

☆ Leverarm & Boresight values result in systematic error and not random error.

WHY?

Because whatever these values are they will remain constant during the entire flight. They remain fixed. Even if they are changing, they will change in a certain fashion only.

That's why they return in systematic error and not random error.



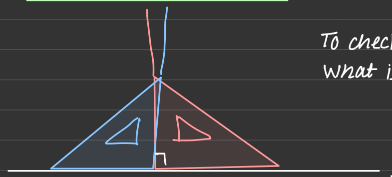
We consider this entire system including GNSS, IMU & scanner to be a RIGID BODY. WHY?

Because the relative position and orientation of the GNSS, IMU and scanner components are fixed w.r.t. each other.
The entire system moves with the same acceleration and velocity and there is no relative movement b/w the sensor, IMU & GNSS.

CALIBRATION

- measurement of d_x, d_y, d_z is called calibration.
- Two ways
  - Measure using total station
  - CAD Drawing

PRINCIPAL OF REVERSAL — In adjustment computation

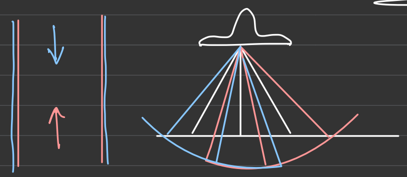


To check whether set square angle is 90° actually?
What is simplest method?

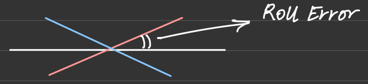
Reverse and see if both lines → overlap or not !!

- Fundamental Principle in adjustment computation.
- To check accuracy of measurement & computation in a network adjustment.
- If set of measurements is made in two directions, algebraic sum of measurements in one direction should be equal to algebraic sum of measurements in the reverse direction.
- If the two sums not equal — error in measurement /computation.

Calibration Approach — In situ (Roll Calibration)



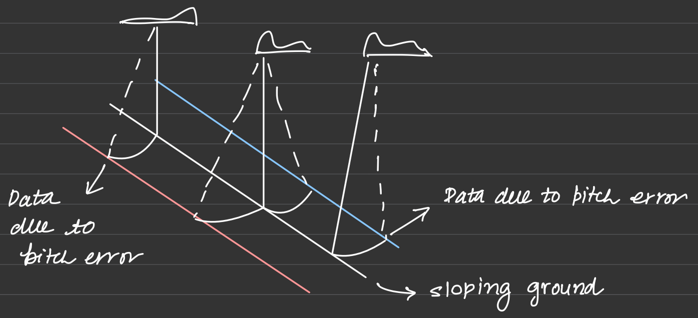
This tells instrument



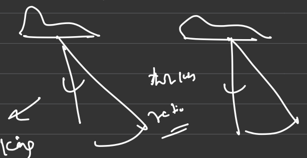
Apply correction to α..

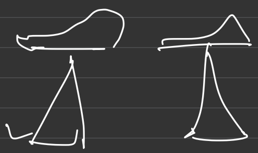
Again do calculation

Calibration Approach — In situ (Pitch Calibration)



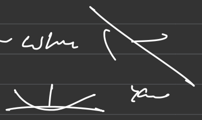
sloping surfaces →





sloping ground ~ when

where not

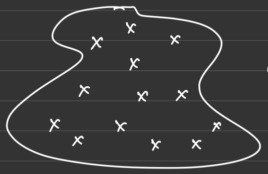


Calibration Approach – In situ (Yaw Calibration)

- Data on opposing 100% flight lines over a specific point.
- Compare translation and determine correction

Field Calibration using GCP

- Identify the GCP – difficult – in case of LiDAR Data
- LiDAR may not hit the ground control point.
- But with large number of GCPs, we can do it.

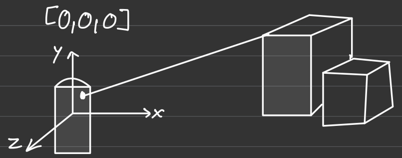


(xyz) — GNSS

(xyz)' — LiDAR Data

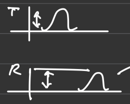
Why? — Data [illegible]

TERRESTIAL LASER SCANNER



- Why minimum range ?

— If the range is too short then the intensity of return pulse is too intense that it saturates the sensor and it fails to work.



→ if detector near → return signal too high

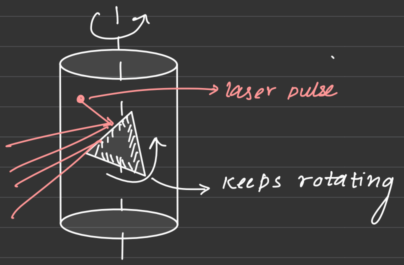
Analogy: Eye gets saturated by sun's rays.

- Why maximum range ?

— Power of instrument — constant

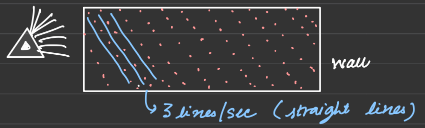
— farther laser travel — more power it requires

- Dual Axis Compensator – sensor – CE331 – total station
- Total Station can be rotated about horizontal.
- RIEGL – Radio wave – to transfer data



- rotating multi-facet mirror
  - has got multiple faces
  - capture data at different directions

- scan speed – 3 lines/sec to 240 lines/sec.



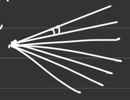
data generated is like this in the form of lines.

- angle measurement resolution

$$2\cdot52\ \text{arc sec} = 2\cdot52 \times \frac{1}{3600}^{\circ}$$

means scanner measure angles with a resolution of 2·52 arcsec it affects the accuracy and precision of data.

- angular step width
  0·0007° to 0·6°

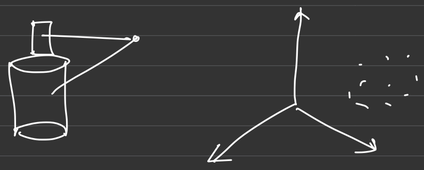


- For vertical scan, $\frac{\text{Angle Measurement}}{\text{Resolution (arcsec)}} = \frac{\text{Angular step Width (degrees)}}{3600}$

- Only a part of mirror facets can be used for measurement.

0·5MHz
PRF
1·2MHz

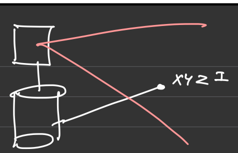
- XYZ — color — Location of orientation of axis
  R G B

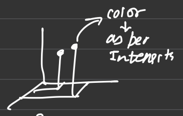


X,Y,Z, I, R,G,B

RGB coloring of Point cloud

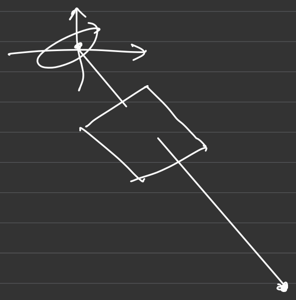
XYZIRGB



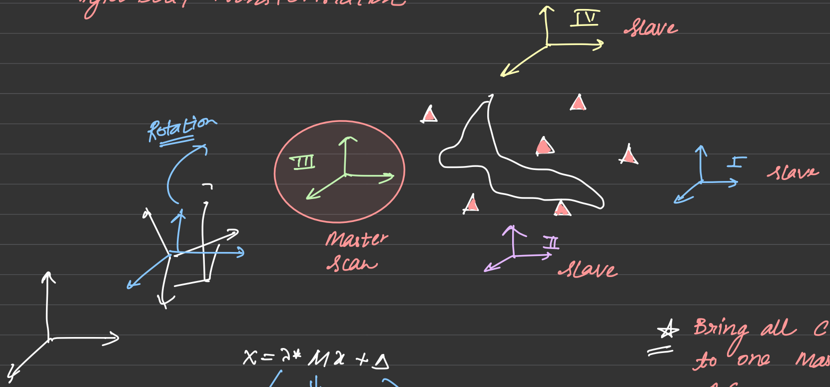


Point Color

Image



Rigid Body Transformation



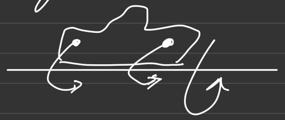
$$X = \underbrace{\lambda}_{\text{Scaling factor}} * \underbrace{M}_{\text{Rotation}} x + \underbrace{\Delta}_{\text{Translation}}$$

★ Bring all C.S. to one Master C.S.

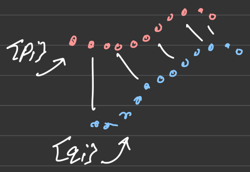
x X

Why only 2 points cannot be used?

Bcoz the point cloud is still rotating about a axis

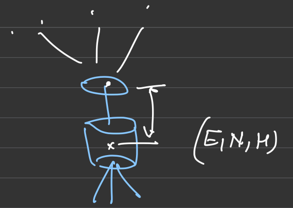


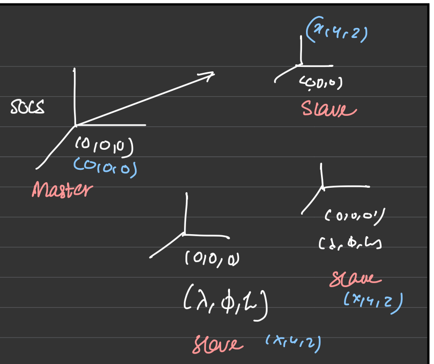
ICP (Iterative Closest Point)



Merging these two point clouds

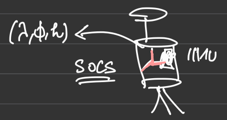
Georeferencing scans



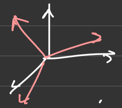


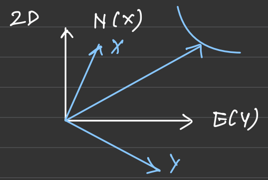
{(0,0,0) — Local — SOCS
(λ,φ,h) — Global — GNSS}

GNSS + Orientation sensors
→ IMU



ω, ϕ, κ

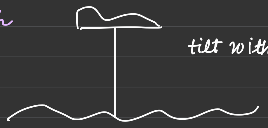




Photogrammetry Measurement using photographs.

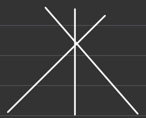
where → XYZ what → Interpretation (Attribute)

Vertical Photograph



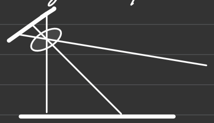
tilt within 3° → called vertical

Oblique



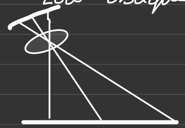
Keeping the camera axis inclined.

High oblique



horizon coming in picture

Low oblique

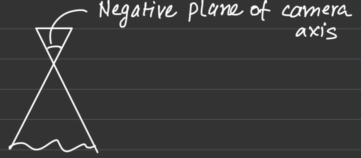


horizon not coming in picture

Terrestial

very common

- Can also find the correct geometry of objects with the photogrammetry.
- UAV fitted with camera → look and capture images of tree.
  - → Know which fruit has ripen?
  - → arm will extend and pluck it.



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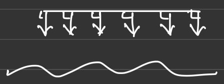
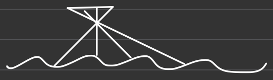
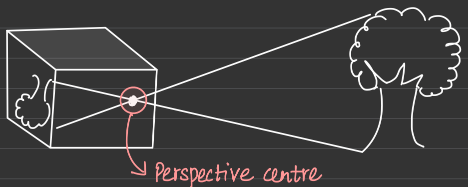
orthographic projection

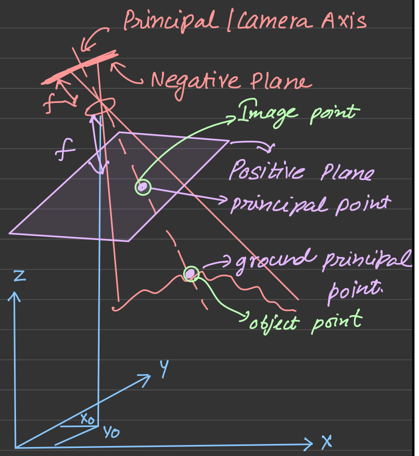


perspective projection



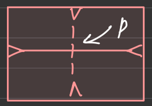
Pinhole camera





- positive plane → an imaginary plane
  ↳ very often talk about it.

Fiducial Marks



marks on photo
to locate principal point
↳ not now, earlier done.

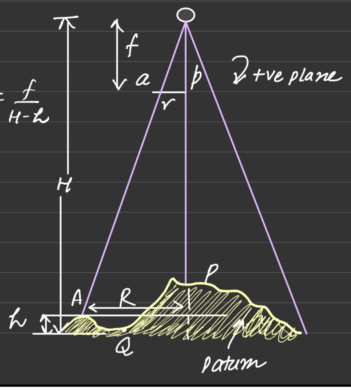
Scale of a Vertical Photograph

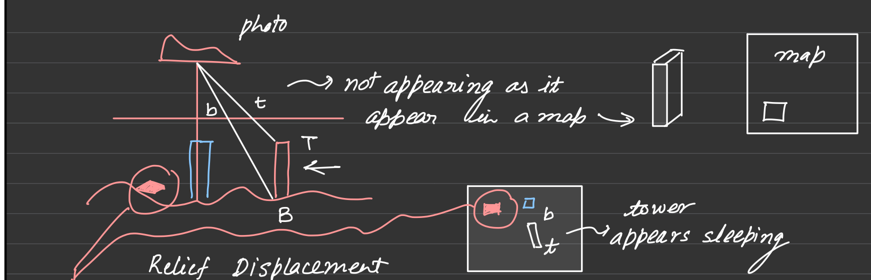
- $\text{Scale} = \frac{\text{Distance on photograph}}{\text{Distance on ground}} = \frac{d\,\text{photo}}{d\text{ground}}$
- Different scale photo — depending upon resolution
- map — scale is uniform — orthographic projection
- photograph — scale is not uniform across one photograph

$$S = \frac{r}{R} = \frac{f}{H-h}$$

$$\boxed{S = \frac{f}{H-h}}$$

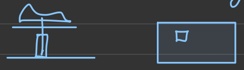
$$S = \frac{r}{R} = \frac{f}{H-h}$$

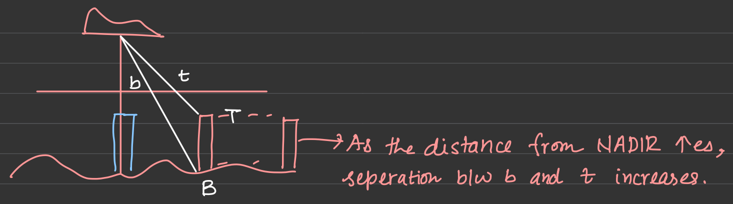




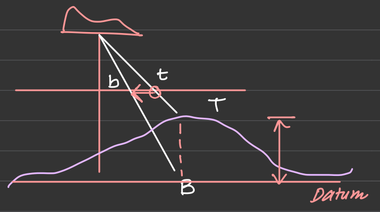
Relief Displacement
or
Displacement due to height.

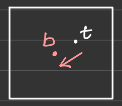
- Whenever objects have got height ⟶ there will be distortions and it will be appearing as sleeping in the image.
- If object is located at the centre of the photograph, at the principle point of the photograph → There is no distortion.





→ As the distance from NADIR ↑es, seperation b/w b and t increases.





Displaced in orthographic photo.

↓

For all points, there will be displacement when the object is at some height from datum.

Relief Displacement

$\Delta OTT'$

$$\frac{r}{R} = \frac{f}{H-h}$$

$$rH - rh = fR \quad ①$$

$\Delta OBB'$

$$\frac{r-d}{R} = \frac{f}{H}$$

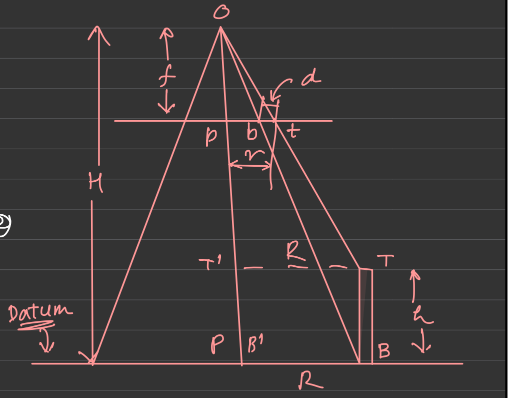
$$rH - dH = fR \quad ②$$

$$rh = dH$$

$$h = \frac{dH}{r}$$

$$\boxed{d = \frac{rh}{H}}$$

$$d \propto r$$
$$\propto h$$
$$\propto 1/H$$



- For satellite photographs
  H → very large
  ⇒ d → very small.
  Appears to be orthophoto bcoz quantum of relief displacement is very low.

Orthophotograph

- A orthophograph — a photo with no relief displacement.
- All the relief displacement are corrected.

Stereo photo

- Something like our eyes —

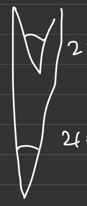
two cameras

Two 2D images

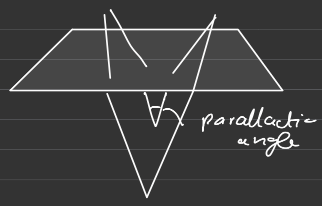
overlap. >60%.

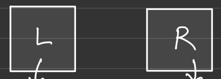
Stereo Pair

Min stereo



max stero





fed to left eye

fed to right eye

Merge to create 3D image.

- same concept is in 3D movie
- Anaglyph → 3D comics

Red → merge ← Blue

- Anaglyph. → 3D comics →
- Polarised Glasses. → left — right → merge

↓ 3D movies.

Red → merge ← Blue

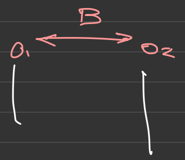
permit only blue image

permit only red image

Incomplete Notes :→ Take notes from others.

For any point 'A' parallax is known as $P_a = c + x_a$

Derivation



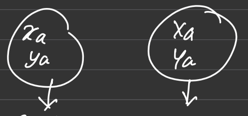


Photo coordinate. Ground coordinate

Derivation — Very well in the video — Watch it!

$P_A = x_a - x_a'$ (parallax)

$\boxed{H, B, f}$? How

$h_a = H - \frac{Bf}{P_a}$ for any general i.

$h_b = H - \frac{Bf}{P_b}$

$h_a - h_b = \frac{Bf}{P_b} - \frac{Bf}{P_a} = Bf\left(\frac{1}{P_b} - \frac{1}{P_a}\right)$

$h_a - h_b = \frac{Bf}{P_a P_b}(P_a - P_b)$

$h_a - h_b = \frac{(H - h_b)}{P_a}(P_a - P_b)$

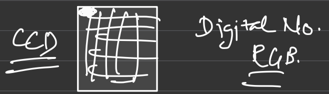
$$\boxed{h_a - h_b = \frac{(H - h_b)}{P_a} \Delta P}$$

ΔP → diff. in parallax.

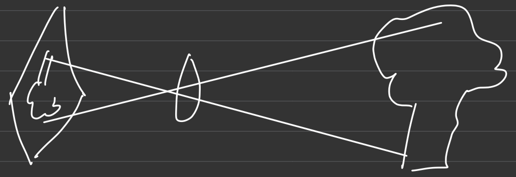
same eqn

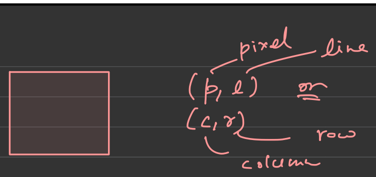
Digital Photogrammetry

CCD Array → charged coupled Device.

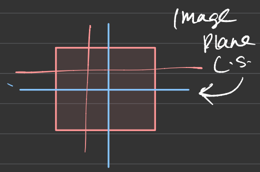


How Image is taken?

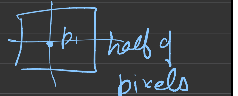


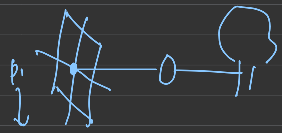


Pixel Coordinate System:-

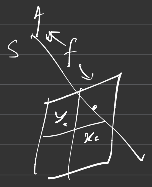


Fiducial Marks → New cameras fiducial marks are not there.

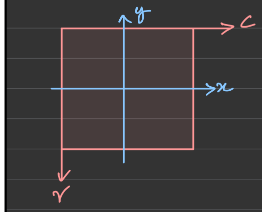


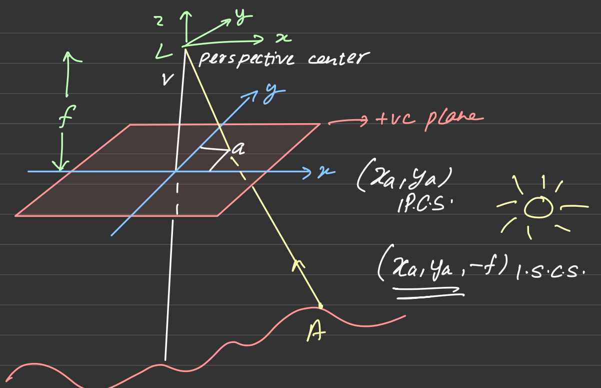


principal point in photograph



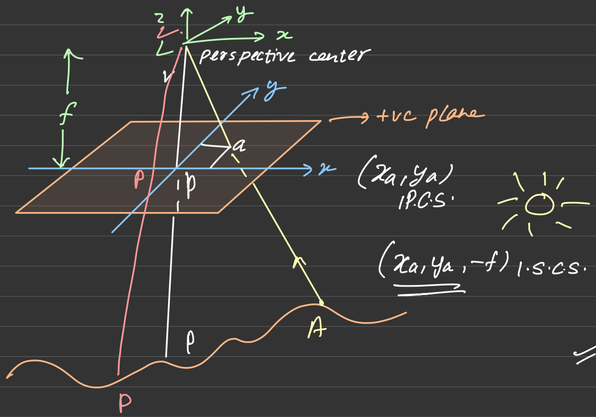
(x_0, y_0, f)
mm mm





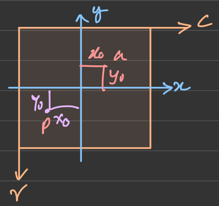
(x_a, y_a) IPCS.

$(x_a, y_a, -f)$ I.S.C.S.



(x_a, y_a) IPCS.

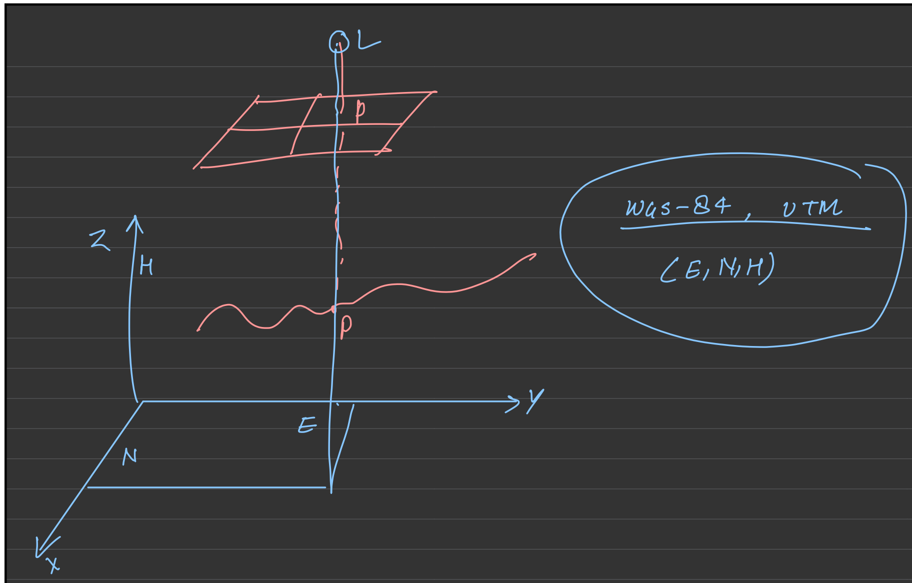
$(x_a, y_a, -f)$ I.S.C.S.



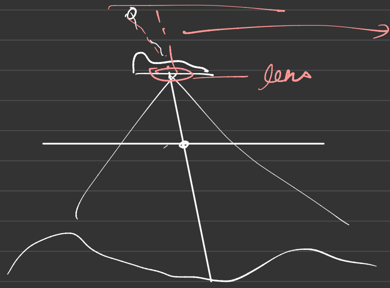
(x_a, y_a) IP

$= ((x_0 + x_a), (y_0 + y_a), -f)$

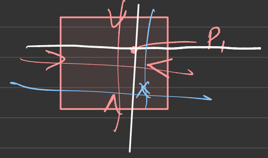
I.S.



Interior Orientation → Internal Geometry at the time of taking photographs.



→ Due to distortion in lens.

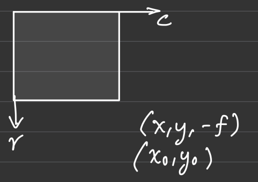


$$\left.\begin{matrix}(c,r)\\(p,l)\end{matrix}\right\} = (x,y)$$

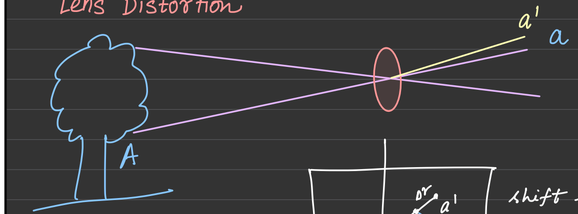
Interior Orientation → Internal Geometry at the time of taking photograph.

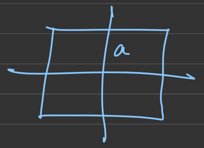
Camera → take the photograph → What was principal point.
→ What was focal length.
→ Is there any distortion in camera lens. What are those distortions.

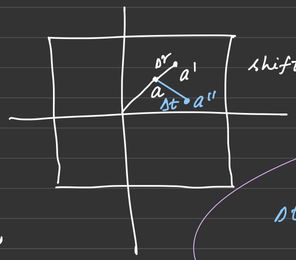
↳ All the internal geometry is determined through the process of camera calibration.



Lens Distortion







shift in the image point due to radial distortion = Δr

Radial Distortion → determine Δr for each pixel in image.
↳ very much there

Δt → Tangential Distortion
↳ very insignificant.

$$\boxed{\Delta r = K_0 r + K_1 r^3 + K_2 r^5}$$

↑ Given by camera manufacturers.

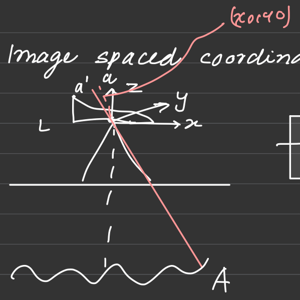
Apply correction.

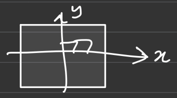
Apply correction
↓
Find original position.

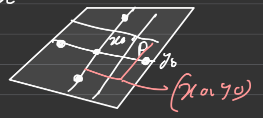
Interior Orientation Parameters → (x_0, y_0) shift of principal point from image center
→ focal length
→ lens distortion parameters
→ no. of pixels, line/length of image

If we know all of these → For any pixel → Get coordinates as
(c, r) or (p, l) $\quad (x_a, y_a, -f)$
$\quad \downarrow x_0 \quad \downarrow y_0$

Image spaced coordinate system

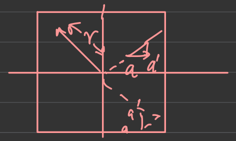






f
$x_0 \quad y_0$
P, L

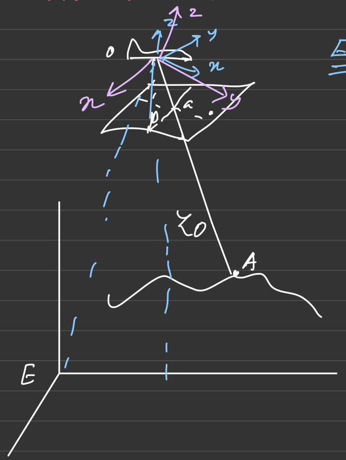
$$\begin{bmatrix} x'_a \\ y'_a \\ -f \end{bmatrix}$$



x_a'
y_a'

Distortion :-
$\Delta r = \underline{K_1} r + \underline{K_2} z^3 + \underline{K_3} r^5 - \cdots$
For every camera these parameters are given.

Exterior Orientation



E.O.
↳ meaning → In ground there is a coordinate corresp. to origin of camera. . . . . .
We want to know at that time when the photo was taken, where is camera and what is orientation of the camera.

$\begin{pmatrix} X_0, Y_0, Z_0 \\ \omega, \phi, K \end{pmatrix}$
↓ elements of Exterior Orientation

$\vec{EO}_a + \vec{OA}_a = \vec{EA}_a$

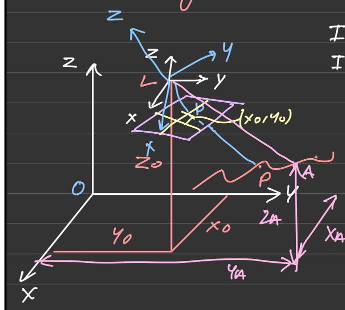
$\vec{OA}_a = \vec{EA}_a - \vec{EO}_a$

$K \cdot \vec{Oa}_a = \vec{EA}_a - \vec{EO}_a$

$K \cdot \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix}$

Collinearity Equation

→ In VideoLecture → Derivation.



I.P.
I.S.

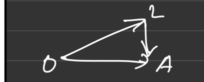
$(x_a, y_a)_{I.P.} \quad ((x_a - x_0), (y_a - y_0), -f)_{\underline{\underline{I.S.C.}}}$

O.S. $\quad A \to (X_A, Y_A, Z_A)_{O.S.} \quad L \to (X_0, Y_0, Z_0)_{O.S.}$

$\vec{OA} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \qquad \vec{OL} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}$

$\vec{La}_{I.S.} = s \cdot \vec{LA}_{I.S.} \qquad (\omega, \phi, K)$

$\begin{bmatrix} x_A - x_0 \\ y_A - y_0 \\ -f \end{bmatrix} = s \cdot \vec{LA}_{I.S.}$



$\vec{OL} + \vec{LA} = \vec{OA}$

$$\boxed{\vec{LA}_{O.S.} = \vec{OA}_{O.S.} - \vec{OL}_{O.S.}}$$

→ Rotation Matrix.
$\vec{LA}_{I.S.} = M \cdot \vec{LA}_{O.S.}$

$R_x(\omega), R_y(\phi), R_z(K) = \begin{bmatrix} m_{11}, & m_{12}, & m_{13} \\ m_{21}, & m_{22}, & m_{23} \\ m_{31}, & m_{32}, & m_{33} \end{bmatrix}$

$\begin{bmatrix} x_A - x_0 \\ y_A - y_0 \\ -f \end{bmatrix} = s \cdot \begin{bmatrix} m_{11}, & m_{12}, & m_{13} \\ m_{21}, & m_{22}, & m_{23} \\ m_{31}, & m_{32}, & m_{33} \end{bmatrix} \cdot \begin{bmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{bmatrix}$

① $x_A - x_0 = s \cdot [\, m_{11}(X_A - X_0) + m_{12}(Y_A - Y_0) + m_{13}(Z_A - Z_0) \,]$
② $y_A - y_0 = s \cdot [\, m_{21}(X_A - X_0) + m_{22}(Y_A - Y_0) + m_{13}(Z_A - Z_0) \,]$
③ $-f = s \cdot [\, m_{31}(X_A - X_0) + m_{32}(Y_A - Y_0) + m_{33}(Z_A - Z_0) \,]$

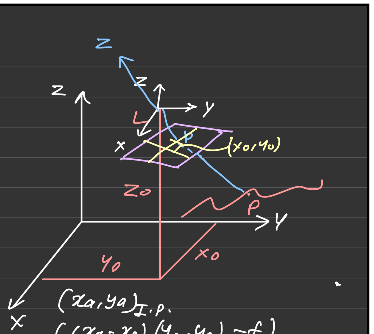
①/③ ⇒ $x_A - x_0 = -$
②/③ ⇒ $y_A - y_0 = -$

Collinearity Equation (In class)

$$K \cdot M \, \vec{Oa}_{ISCS} = \vec{EA_G} - \vec{EO_G}$$

$$\vec{O_G^a} = k' \begin{bmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{bmatrix}$$

$$\begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} = k' \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{bmatrix}$$



$(x_a, y_a)_{I.P.}$

$((x_a - x_0), (y_a - y_0), -f)_{I.S.C.}$

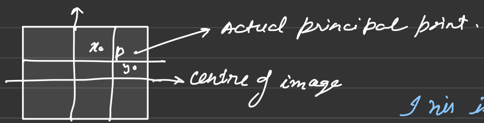
$$x_a = k' \left[m_{11}(X_A - X_0) + m_{12}(Y_A - Y_0) + m_{13}(Z_A - Z_0) \right]$$

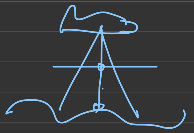
$$y_a = k' \left[m_{21}(X_A - X_0) + m_{22}(Y_A - Y_0) + m_{23}(Z_A - Z_0) \right]$$

$$-f = k' \left[m_{31}(X_A - X_0) + m_{32}(Y_A - Y_0) + m_{33}(Z_A - Z_0) \right]$$

$$x_a = \frac{-f[-]}{[-]}$$

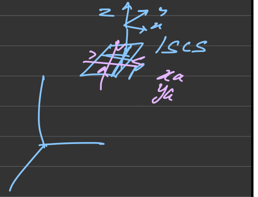
$$y_a = \frac{-f[-]}{[-]}$$





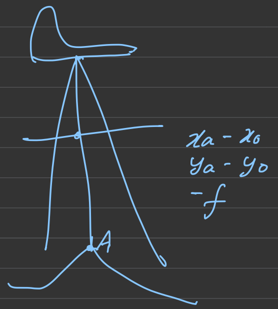
This is relating

$\vec{Oa} = k \cdot \vec{OA}$



It is relating coordinates observed in a photograph with the ground coordinates of panopoints.

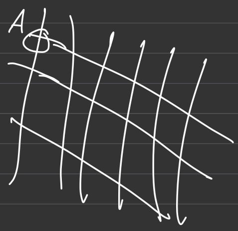
↳ Collinearity Equation.



$x_a - x_0$
$y_a - y_0$
$-f$

everything known in eq^ns except X_A, Y_A, Z_A } ground coordinates

1 point → 2nd eq^n of collinearity → 3 unknowns (X_A, Y_A, Z_A)

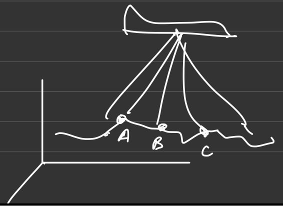


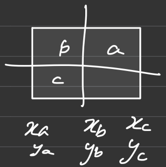
IO x_0 y_0 $-f$?

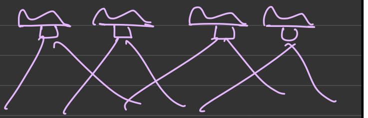
X_A
Y_A
Z_A

[Camera Calibration]

- 10 Parameters — don't change over time — remain same — change over 3-4 yrs only.
- EO Parameters — changing every time — X_0, Y_0, Z_0 ω, ϕ, κ.



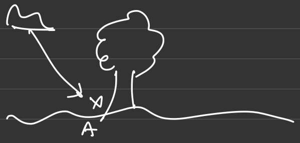




Photogrammetry — Limitation w.r.t. Laser scanning

In case there is a point under a tree.

For Lidar → just need one laser beam to reach that point and we'll be able to compute x, y, z.



For Photogrammetry

↳ same point may not be visible from the 2nd photo.

↓

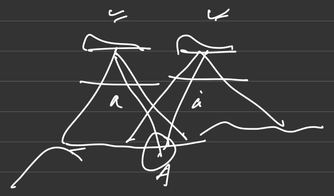
Condition / Assumption of Photogrammetry.

↓

Neccesary

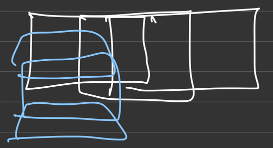
For large non-textured places

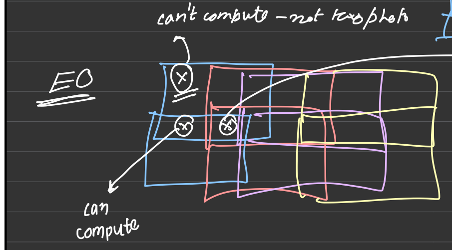
↳ Difficult to use photogrammetry.



IO ✓
EO ✓ } known.

60% overlap ✓ photos





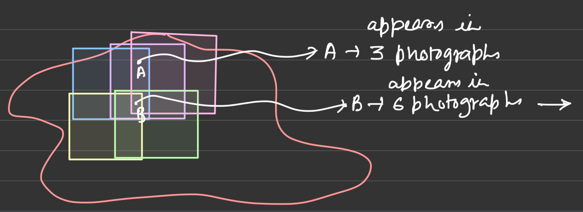
4 photos it is coming

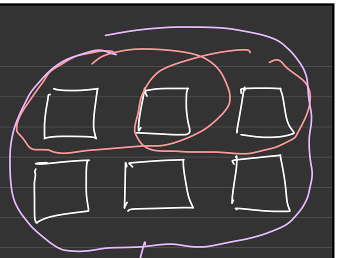
$4C_2$ ways of doing

↓

which answer is correct.

· we treat entire thing as one system.





Taking Multiple Photographs to determine coordinates of a pt. on ground

↓

called Bundle Block Adjustment.

↓

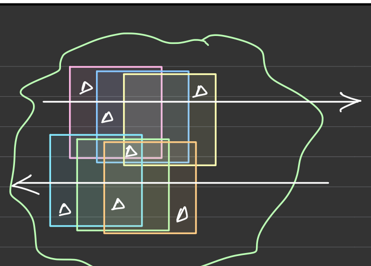
Do space intersection for all the photos

↓

Do the least square.

12 collinearity.

X_B, Y_B, Z_B



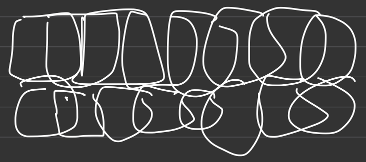
100 ×42

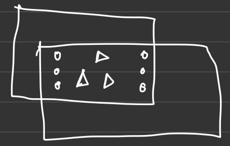
(x,y)

Idea of Bundle Block Adjustment.

EO → for all photos.
IO

Bundle block adjustment

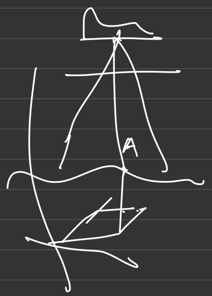




Δ GCP → Ground Control Point

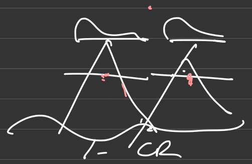
∘ Tie point → we know only image space coordinates.
→ we don't know ground coordinates.

$$A^* \begin{bmatrix} x_a \\ y_a \\ -f \end{bmatrix} \text{ as well as } \begin{bmatrix} x_A \\ y_A \\ -f \end{bmatrix},$$



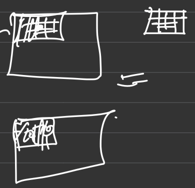
- Tie point – common in both two photos
- many points. corner of the road.

well.



Tie point.

Automatic Method :-



can compute mean of each [illegible] of photo

3X3
5X5
7X7 or whatever

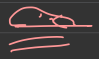
$\bar{g}$, $\bar{\sigma}$

mean std-dev.

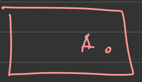
pixel - mean / std.

Correlated Method

SHIFT | SURF Algorithm.



Finally →



$x_a, y_a, -f$]

1GCP → 4 eqns.

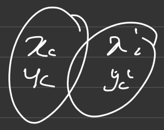
3GCPs → 3X4 = 12 photo.

6GCP → 36 eqn generate

[illegible]

How many unknown are there!

To know eq [illegible]



External Orientation Parameters

EO

Total no. of unknowns = For each of the p

Unknown parameter..
Total 30

} Total 30 unknown

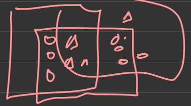
3GCP – ground
\+
photo

4 degree of freedom each.
<7

Not only two → one more photo can also be there..

- GCP → 4x 4 = 16
- TP → 16 +18 = 50

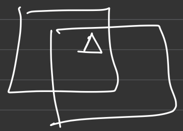
DS



HOW many GCPs?

Write coordinates?

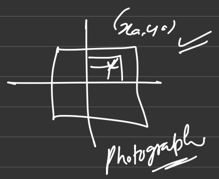
3D GCP

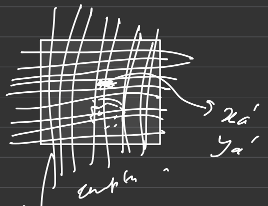


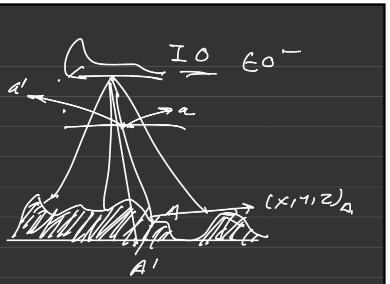
2·n redundant.

Ortho Photo Generation: →

DEM ⇐ ht. from datum Knows





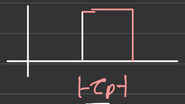


All the points
will shift

New Image – ortho Image

23.5 /45

④



PRF = 100KHz

$t = \frac{2H}{c}$

time available for one pulse $= \frac{1}{PRF}$

$$\boxed{\frac{1}{PRF} = \frac{2H}{c}}$$

Additional
time τ_p
↓
to come to ground
and go

$\frac{1}{PRF} = \frac{2H}{c} + 2\tau_p$

H = 1497m

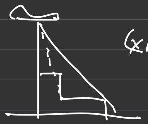
waiting time
2ns

$\frac{1}{PRF} = \frac{2H}{c} + 2\tau_p + 2ns$
↓
FOS

⑤

A) B ✓
B) A ✓
C) B ✓
D) Both

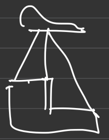
⑥ ① R1 ≈ R2



(x,y) same
$z_1 \neq z_2$

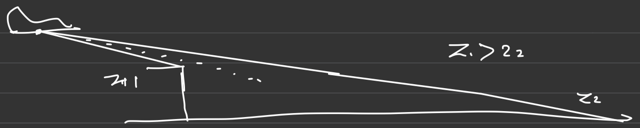
(d) $x_1 \neq x_2$, $y_1 \neq y_2$, $z_1 = z_2$

(e)



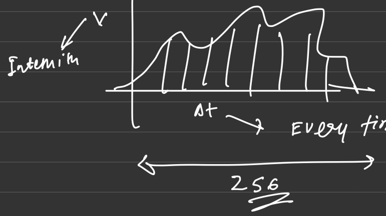
(f)

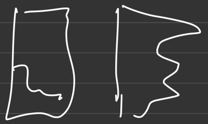
(f)



(g) $\propto$

Q2 Gaussian Pulse





Every time we are getting intensity of the

② $\Delta R \propto \Delta t$ (Pulse)

ΔR → Resolution

$\Delta R \propto \lambda \cdot \Delta\phi$ (C·W)

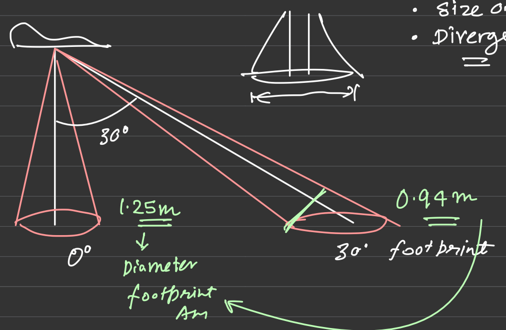
$\Delta R = \frac{1}{4\pi}\left(\frac{c}{f}\right)\Delta\phi$

$\frac{c}{f}$ → wavelength

$\Delta\phi$ → Phase measurement

Derivation

③



- Size of the footprint
- Divergence

↳ $IFOV = 2.44\frac{\lambda}{D}$ — nm, μm

↓ ° (Degree)

$$\boxed{P_r = \frac{\rho \cdot M^2 \cdot D_r^2 \cdot D_{tar}^2 \cdot P_Z}{4R^2(R \cdot \gamma + D)^2}}$$

ρ → 80%

M^2 → 0.01

D_r^2 → 20cm

R → Range

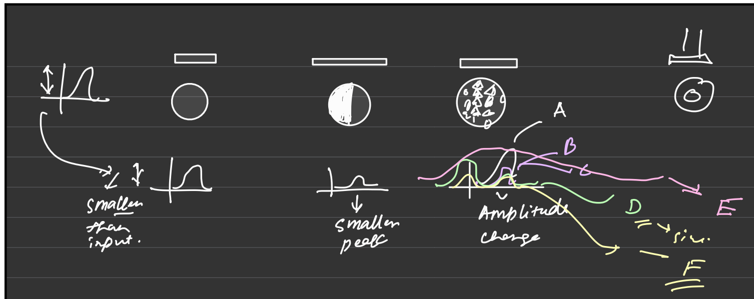
γ → Divergence → in Radian

D_{tar}^2 → Not given → 40% of area is targent. → Area

P_Z → 800

D → Diameter of transmitting object.

$P_r = 4.117 \times 10^{-10} \omega$



Pulse → P_{peak} → high → for same average energy

↓

Pulse can generate higher peak

↓

1000 x order higher than CW wave.

↓

In C.W. size of instrument become very high.

can travel more large distances