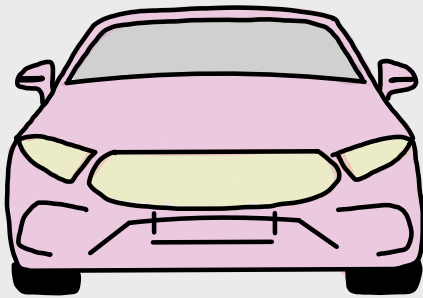
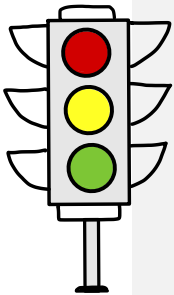


# CE781A

## TRAFFIC SIMULATION

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Aman

### CE781A Traffic Simulation

Traffic Flow Dynamics — By Martin Triebler  
(Matlab/Python)

{ Que → 40% }  
{ MS/les → 60% }

Settings 0.4 Thick  
H1 → H2 → H3 → H4 →

8 Jan 2024 (Mon)

#### Computer Simulation

mimic real world traffic scenario setting in the computer using mathematical model.

#### Why simulation?

- To design and operate traffic system
- Traffic Flow: highly complex and non-linear behaviour (multiple vehicles)
- Check new traffic facility eg. Bus-bay, intersection, traffic signal, different settings / scenario that are difficult to do in field.

#### Types of Traffic Simulation

##### 1. Macroscopic Simulation

- Represents collection of vehicles
- Locally aggregated vehicles
- Useful for understanding traffic flow on a larger scale

##### Models

- First order (LWR) model
- Second order model

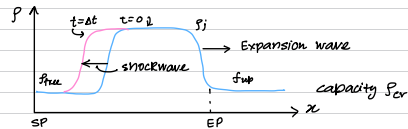
##### Models using

- Fluid Flow Analysis
- Gas Kinetic Theory
- Convert microscopic to macroscopic model

##### Numerical Methods

- (FD, EV, FE)
- Boundary cond<sup>n</sup>
- Initial cond<sup>n</sup>
- $Q = \frac{1}{\tau}$
- $f = \frac{1}{\tau}$

Disordered traffic → Indian traffic



##### 2. Microscopic simulation

Models individual vehicles

##### Models

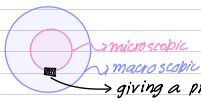
- Car Following Model
- Lane Changing Model

##### Commercial Softwares

- VISSIM (Germany)
- AIMSIM (Spain)
- TransModeler (USA)

### 3. Mesoscopic Simulation

Combines partly aggregate (macro) and partly individual (micro)



giving a proper interface is challenging

#### Random Event

Before conducting the experiment, you don't know the outcomes  
But you know the possible outcomes eg. tossing a coin

#### Random Number

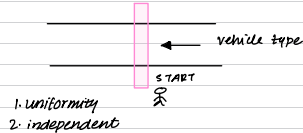
Real Number lies b/w 0 to 1

$$0 < R < 1$$

Generate vehicle type using random numbers

'rand' built-in function to generate them

- $r_1 = 0.8$  Bus
- $r_2 = 0.002$  TW
- $r_3 = 0.46$  Car
- $\vdots$
- $r_{100} = \dots$



- uniformity
- independent

	cumm	cumm fraction
TW	25	0.25
Car	25	0.50
Auto	25	0.75
Bus/Truck	25	1.00
	100	

RN  $RN_1 = 0.89 \rightarrow$  Bus  
 $RN_2 = 0.46 \rightarrow$  Car

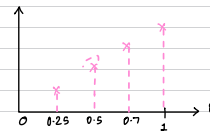
10 Jan 2024 (Wed)

Blackbox

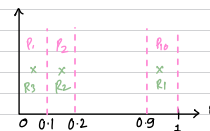
'rand' in MATLAB — good random no. generator

Two qualities

- uniformity — equal probable everywhere  $\rightarrow$  eg. toss coin  $P(X=H) = 0.5$   
 $\{H, T\}$   $P(X=T) = 0.5$
- independence — doesn't get affect from previous event.



R — continuous random variable  
 $P(0.25) = P(0.5) = P(0.75) = 0$   
meaningless to talk abt it.



R — continuous random variable  
10 Intervals  
 $P_1 = P_2 = P_3 = \dots = P_{10}$   
Uniformity  
Interval size — same

- $P_1 (0 < R < 0.1)$
- $P_2 (0.1 < R < 0.2)$
- $P_3 (0.2 < R < 0.3)$

#### Mid-square method (By John von Neumann, CS professor)

##### Algorithm

- Initialize with 4 digit number ( $Z_0$ ) iterative method
- Square ( $Z_0^2$ ) i.e. 8 digit number
- Consider middle 4 digits ( $Z_1$ )
- Introduce decimal point before  $Z_1$  ( $0.Z_1$ )
- Repeat step 2 using  $Z_1$  (not  $0.Z_1$ )

eg: — consider 2 digit no for simplicity

Iteration 1	Iteration 2	Iteration 3
1. $Z_0 = 77$	1. $Z_1 = 92$	1. $Z_2 = 46$
2. $Z_0^2 = 77^2 = 5929$	2. $Z_1^2 = 8464$	2. $Z_2^2 = 2116$
3. $Z_1 = 92$	3. $Z_2 = 46$	3. $Z_3 = 11$
4. $R_1 = 0.92$	4. $R_2 = 0.46$	4. $R_3 = 0.11$

After some iterations, again  $Z_2 = 46$  will come up. Thus, there is a cycle that will repeat in method. Always, this cycle will be there. In computers, cycle is unavoidable.

mid sq. method — useful for generating number sequence  
cycle period ( $Z_0$  to  $Z_{i-1}$ ) → cycle repeats

generate larger cycles, how?  $Z_0 =$  seed value (initial value)  
cycle length depends on  $Z_0$ .

- computer always generate dependent random variable.
- vanishing random variable.

**pseudo random variable** — computer generated random variable (artificial)  
 — not pure because cycle is unavoidable and dependent

**Vanishing RV**

$X_0 = 73$	32	02	
$Z_0 = 5329$	1024	0004	
$Z_1 = 32$	02	00	random variable now vanishes....
$R_1 = 0.32$	0.02	0.0	

For a good RV generator

1. ↑ cycle length
2. avoid vanishing
3. uniformity

**Linear congruential method**

$$x_{i+1} = (ax_i + c) \text{ mod } m$$

$a$  - multiplier  
 $c$  - increment  
 $m$  - mod - remainder  
 $x_0$  - seed value  
 $R_{i+1} = \left\{ \frac{0}{m}, \frac{1}{m}, \frac{2}{m}, \dots, \frac{m-1}{m} \right\}$  you can choose to divide by  $(m-1)$  as well  
 $R_{i+1} = \{0, \dots, 1\}$  Random number

Assumption:  $m > 0$ ,  $a < m$ ,  $x_0 < m$ ,  $c < m$

eg:  $x_1 = (17 \times 73 + 43) \text{ mod } 100 = 502 \text{ mod } 100 = 2$   $R_1 = 0.02$   
 $x_2 = (17 \times 2 + 43) \text{ mod } 100 = 77$   $R_2 = 0.77$   
 $x_3 = (17 \times 77 + 43) \text{ mod } 100 = 52$   $R_3 = 0.52$

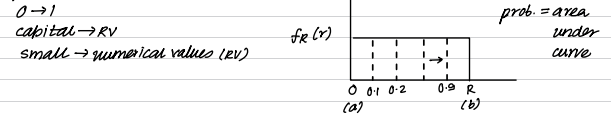
cycle — max cycle time possible =  $m$   
 choose  $m = 2^{31}$  or  $2^{49}$ , based on your computer storage  
 this guarantee large cycle length

$m = 100$   
 $x_{i+1} = \{0, 1, 2, \dots, 99\}$   
 $R_{i+1} = \{0, 0.01, 0.02, \dots, 0.99\}$  → continuous R.V

**continuous RV**

1. uniformity → To ensure uniformity b/w  $(a, b)$  we choose uniform distribution
2. independence

pdf of uniform distribution



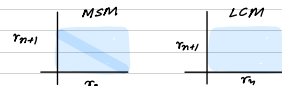
Area under the curve = total probability  $R \rightarrow [0, 1]$   
 width  $\times$  height = 1  
 $(b-a) \times f_R(a) = 1$

$$f_R(x) = \frac{1}{b-a} \text{ pdf } \sim \text{uniform } (a, b)$$

$\sim$  mean  $R = \frac{a+b}{2} = 0.5$   
 $\sim$  variance =  $\frac{(b-a)^2}{12} = \frac{1}{12}$

$R \rightarrow$  uniform  $(0.5, 1/12)$  distribution

$m = 6075$  } LCM is better than MSM  
 $a = 106$   
 $c = 1205$   
 $R_n \rightarrow (5000)$   
 $R_m \rightarrow (5000)$



1. vehicle type
2. time headway
3. speed

$t_1 = 10 \text{ sec}$   
 $t_2 = 2.5 \text{ sec}$   
 $t_3 = 186 \text{ sec}$

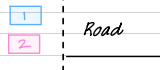
**Random Variable**

PDF  $x = \frac{1}{\lambda} \ln(R)$

17 Jan (Wed)

**Inputs**

1. vehicle type (RN)
2. time headway (exponential distribution)



**Speed**

- 1) Desired speed
  - 2) Entry speed
- speed ~ Normal Distribution

**Normal Random Variable**

Normal PDF  $f_x(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$   $(\mu, \sigma)$

Standard Normal PDF  $f_x(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$   $(0, 1)$

$$F_x(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \rightarrow \text{non-elementary function}$$

Assume two standard normal RN  
 $X \sim N(0,1)$   $Y \sim N(0,1)$

$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$  pdf X  
 $f_y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$  pdf Y

$X$  &  $Y$  independent  $\Rightarrow \rho = 0$  (correlation coeff)

$\rho = 0 \Rightarrow X$  &  $Y$  independent

Joint PDF of  $X$  &  $Y$

$$f_{X,Y}(x,y) = f(x) f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} = \frac{1}{2\pi} e^{-\frac{(x^2+y^2)}{2}}$$

→ legitimate PDF?

**CDF**

$$F_{X,Y}(x \leq x', y \leq y') = \int_{-\infty}^{x'} \int_{-\infty}^{y'} \frac{1}{2\pi} e^{-\frac{(x^2+y^2)}{2}} dx dy$$

$$= \frac{1}{2\pi} \int_{-\infty}^{x'} \int_{-\infty}^{y'} e^{-\frac{(x^2+y^2)}{2}} dx dy$$

$X, Y \rightarrow$  cartesian coordinate system  
 Changing to Polar coordinate system

$$\left. \begin{aligned} r^2 &= x^2 + y^2 \\ \tan^{-1}\left(\frac{y}{x}\right) &= \theta \\ r dr d\theta &= dx dy \end{aligned} \right\} \begin{aligned} R, \theta & \quad 0 \leq R \leq \infty \\ & \quad 0 \leq \theta \leq 2\pi \end{aligned}$$

$$F_{R,\theta}(R \leq r', \theta \leq \theta') = \frac{1}{2\pi} \int_0^{r'} \int_0^{\theta'} e^{-r^2/2} r dr d\theta$$

$$= \frac{1}{2\pi} \int_0^{r'} e^{-r^2/2} \int_0^{\theta'} d\theta r dr$$

$$= \frac{1}{2\pi} \int_0^{r'} e^{-r^2/2} \int_0^{2\pi} d\theta r dr$$

$$= \int_0^{r'} e^{-r^2/2} r dr \quad (\text{let } r' \rightarrow \infty)$$

→ now solvable!

$$F_{R,\theta} = \int_0^{r'} e^{-r^2/2} r dr$$

Put  $s = \frac{r^2}{2}$   $ds = r dr$

$$F_{s,\theta} = \int_0^{s'} e^{-s} ds$$

$$\lim_{s \rightarrow \infty} F_{s,\theta} = -[e^{-s}]_0^{s'} = -[e^{-\infty} - e^0] = 1$$

Thus, we have proved it is a legitimate PDF.

$$F_{X,Y}(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{(x^2+y^2)}{2}} dx dy$$

Objective: we want standard normal random variable.

$x, y \quad f=0$

$f=0$  circle from top  
 $f \neq 0$  ellipse from top

fix  $R$   
 $\theta$  - equally likely  
 $\theta$  follows uniform distribution  
 $0 \leq \theta \leq 2\pi$   
 $\theta = 2\pi u_1$  uniform RV (0,1)

$f_{R,\theta} = \frac{1}{2\pi} \int_0^{r'} \int_0^{\theta'} e^{-r^2/2} r dr d\theta$        $\theta' \rightarrow 2\pi$   
 $r' \rightarrow \infty$

$f_{R,\theta} = \frac{1}{2\pi} \int_0^{r'} e^{-r^2/2} \int_0^{2\pi} r dr = \int_0^{r'} e^{-r^2/2} r dr$

$r^2$  follow exponential distribution

$s = \frac{r^2}{2} \quad ds = r dr$

$F_{s,\theta} = \int_0^{s'} e^{-s} ds = [-e^{-s}]_0^{s'}$

$u_2 = [-e^{-s}]_0^{s'}$

$u_2 = 1 - e^{-s'}$

$u_2 = 1 - e^{-r^2/2}$

$e^{-r^2/2} = 1 - u_2$

$-\frac{r^2}{2} = \ln(1 - u_2)$

$r^2 = -2 \ln(1 - u_2) \quad 0 \leq R < \infty$

$r = \pm \sqrt{-2 \ln(1 - u_2)}$

$r = \sqrt{-2 \ln(u_2)} \quad \theta = 2\pi u_1 \quad u_1 \sim R(0,1)$   
 $R = \sqrt{-2 \ln u_2} \quad u_2 \sim R(0,1)$

$X = R \cos \theta = \sqrt{-2 \ln u_2} \cos(2\pi u_1)$   
 $Y = R \sin \theta = \sqrt{-2 \ln u_2} \sin(2\pi u_2)$

$x, y \sim$  standard normal var.  
 convert back polar  $\rightarrow$  cartesian

Box-muller transformation

$X = \sqrt{-2 \ln u_2} \cos(2\pi u_1)$   
 $Y = \sqrt{-2 \ln u_2} \sin(2\pi u_1)$  } Standard Normal Random Variable

$\mu$        $\sigma$

$V_{car} \sim N(60 \text{ kmph}, 5 \text{ kmph})$   
 $V_{truck} \sim N(40 \text{ kmph}, 3 \text{ kmph})$

$X = V_{car} - \mu_{car} \quad Y = V_{truck} - \mu_{truck}$   
 $\downarrow \quad \sigma_{car} \quad \downarrow \quad \sigma_{truck}$   
 $N(0,1) \quad N(0,1)$   
 SNRV      SNRV

$V_{car} = \mu_{car} + \sigma_{car} \sqrt{-2 \ln u_2} \cos(2\pi u_1)$   
 $V_{truck} = \mu_{truck} + \sigma_{truck} \sqrt{-2 \ln u_2} \sin(2\pi u_1)$

$V_{car} = 60 + 5 \sqrt{-2 \ln u_2} \cos(2\pi u_1)$   
 $V_{truck} = 40 + 3 \sqrt{-2 \ln u_2} \sin(2\pi u_1)$

Disadvantage:  $-\infty < V_{car} < +\infty$   $\rightarrow$  Truncated Normal Distribution

$\{ 50 \leq V_{car} \leq 70 \}$   
 $\downarrow$   
 At end, regenerate this limit.

Log Normal Random Variable

Take its log.

• Install - Automated Driving Toolbox  $\rightarrow$  for lab!  
 Next class - 10M

$\rightarrow$  Model  
 $\rightarrow$  Implementation  
 $\rightarrow$  Data (Sim. Setup)  
 $\rightarrow$  Noise  
 $\rightarrow$  Calibration  $\rightarrow$  everything they calibrate. succeed in some data only not all the data. Parameter reaching optimal value then also do the calibration.

Lab #2

$X = \sqrt{-2 \ln u_1} \cos(2\pi u_2)$   
 $Y = \sqrt{-2 \ln u_1} \sin(2\pi u_2)$

function [output variable] = functionname (input variable)  
 end  
 output variable = function.name (input variable)

---

Input Variables 19 Jan (Fri)

Move the vehicles  
 1) models  
 $\rightarrow$  longitudinal dynamics — Car Following Model  
 $\rightarrow$  lateral dynamics — Lane Changing Model

CAR FOLLOWING MODEL

$\rightarrow$  mathematical formulation  
 $\rightarrow$  coupled ODE

$\frac{dx}{dt} = v$   
 $\frac{dv}{dt} = f(s, v_f, v_l)$

$f \rightarrow$  social force  
 $f$  is not a physical force

$\rightarrow$  Newtonian mechanics  
 $F = ma \quad X$

$\rightarrow$

F      L

1) Newton's third law  
 2) Reaction time

Independent variable  
 1) space gap (s)  
 2) speed of the follower  
 3) Relative speed b/w leader and follower (RV)

$\Delta V = V_f - V_l$

1) Eulerian framework — inside vehicle sit.  
 2) Lagrangian framework — outside veh. fix the position  
 $\rightarrow$  Car Following model

Car following model  
 $\rightarrow$  describes all the driving regimes  
 $\rightarrow$  free flow (empty road)  
 $\rightarrow$  steady state ( $\frac{dV}{dt} = 0$ )  
 $\rightarrow$  dynamic situations

smooth transitions b/w different traffic regimes  $\frac{dV}{dt}$   
 $jerk = \frac{d^2V}{dt^2} = \text{some finite value}$   
 $\downarrow$   
 smooth & continuous

Intelligent Driver Model (IDM)

First principles (

$$\frac{dv}{dt} = \underbrace{\text{free term}} + \underbrace{\text{interactive term}}$$

$$\text{free term} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right)$$

$$\text{interactive term} = -a \left( \frac{s'}{s} \right)^2$$

(congested traffic)

- $a$  = max. acceleration (modal parameter)
- $v$  = current speed of vehicle
- $v_0$  = desired speed of vehicle (modal parameter)
- $s'$  = desired spacing
- $s$  = actual spacing
- $\delta$  = a modal parameter

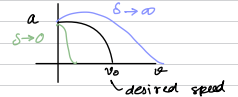
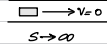
$$s' = s_0 + vT + \frac{v \Delta v}{2 \sqrt{ab}}$$

- $T$  = desired time gap
- $v$  = current speed of veh.
- $\Delta v = v_f - v_l$
- $s_0$  = minimum gap
- $a$  = max. acceleration
- $b$  = comfortable deceleration.

$$\frac{dv}{dt} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right) - a \left( \frac{s'}{s} \right)^2$$



Case-1 Empty Road



$$\frac{dv}{dt} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right)$$

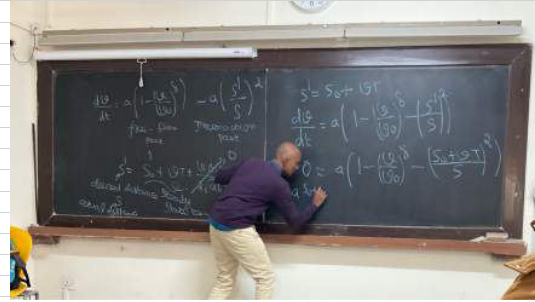
$$v=0 \Rightarrow \frac{dv}{dt} = a \text{ (max acc.)}$$

$$v=v_0 \Rightarrow \frac{dv}{dt} = 0$$

Why  $\delta$ ?  
to control shape of the curve.

24-Jan (Wed)

$$\frac{dv}{dt} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right) - a \left( \frac{s'}{s} \right)^2$$



$$s' = s_0 + vT$$

$$\frac{dv}{dt} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right) - a \left( \frac{s'}{s} \right)^2$$

$$0 = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta - \left( \frac{s_0 + vT}{s} \right)^2 \right)$$

$$\frac{a (s_0 + vT)^2}{s^2} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta \right)$$

$$\frac{(s_0 + vT)^2}{s^2} = 1 - \left( \frac{v}{v_0} \right)^\delta \quad \delta = 4$$

$$v, s \quad \frac{s^2}{(s_0 + vT)^2} = \frac{1}{1 - \left( \frac{v}{v_0} \right)^\delta}$$

$$s^2 = \frac{(s_0 + vT)^2}{1 - \left( \frac{v}{v_0} \right)^\delta}$$

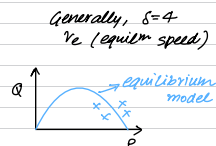
$$s_e = \frac{(s_0 + vT)}{\sqrt{1 - \left( \frac{v}{v_0} \right)^\delta}}$$

Steady state gap

$$p = \frac{1}{p_0 + l}$$

$$p_{avg} = \frac{1}{\text{distance headway}}$$

$$Q = p \cdot v$$



Steady state means  $\frac{dv}{dt} = 0$

$$s' = s_0 + vT + \frac{v \Delta v}{2 \sqrt{ab}}$$

free part = 0  
steady state part = 0

- 1) free flow part
- 2) steady state part
- 3) dynamical part (signal)

$$\frac{dv}{dt} = a \left( 1 - \left( \frac{v}{v_0} \right)^\delta - \left( \frac{s'}{s} \right)^2 \right)$$

$$\frac{dv}{dt} = -a \left( \frac{s'}{s} \right)^2$$

$$\frac{dv}{dt} = -a \left( \frac{v \Delta v}{2 \sqrt{ab} s} \right)^2$$

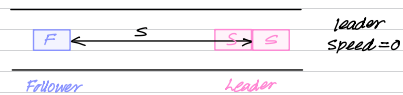
$$\frac{dv}{dt} = -a \frac{v^2 \Delta v^2}{4 ab s^2} = - \frac{v^2 \Delta v^2}{4 b s^2}$$

$$\Delta v = v_f - v_l = 0$$

$$0 \Delta v = v_f = 0$$

$$\frac{dv}{dt} = - \frac{v^2 \Delta v^2}{4 b s^2} = - \frac{v^4}{4 b s^2} = - \frac{b_{kin} v^2}{b}$$

where,  $b_{kin} = \left( \frac{v^2}{2s} \right)$



$$v^2 = u^2 + 2as$$

$$0 = u^2 + 2as$$

$$a = - \frac{u^2}{2s}$$

$$\frac{dv}{dt} = - \frac{b_{kin} v^2}{b}$$

(dynamic part)

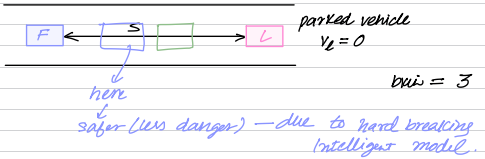
$b$  = comfortable deceleration = 2 m/sec<sup>2</sup>

Case 1  $t = t_1$



$$\frac{v^2}{2s} = b_{crit} = 4 \text{ m/sec} \quad \frac{dv}{dt} = \frac{-b_{crit} \times b_{crit}}{b} = \frac{-4 \times 4}{20} = -0.8$$

overcompensating - too dangerous situation there applying hard braking



$$t = t_1 + \Delta t$$

$$\frac{dv}{dt} = \frac{-3 \times 3}{20} = -4.5 \text{ m/s}^2$$

Case-2

$$\frac{dv}{dt} = \frac{-b_{crit}^2}{b}$$

$$\left. \begin{aligned} b_{crit} &= 2 \text{ m/s}^2 \\ b &= 4 \text{ m/s}^2 \end{aligned} \right\} \frac{dv}{dt} = \frac{-2 \times 2}{4} = -1$$

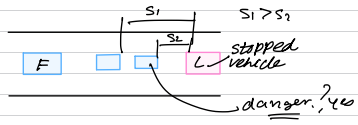
mild braking

t + \Delta t

$$\left. \begin{aligned} b_{crit} &= 4 \text{ m/s}^2 \\ b &= 4 \text{ m/s}^2 \end{aligned} \right\}$$

$$\frac{dv}{dt} = \frac{-4 \times 4}{4} = -4 = \underline{b}$$

Intelligent Driver Model  
↓  
Best model so far)



OPTIMUM VELOCITY MODEL ~ Japanese Model  
Some Drawbacks

FULL VELOCITY DIFFERENCE MODEL ~ some drawbacks } modified

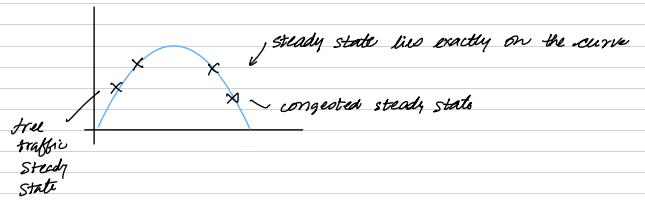
MODIFIED FULL VELOCITY DIFFERENCE MODEL ~ good one, but won't teach (no time)

Steady state means  
↑ max acc → anticipations ↑ → steady state (steady state always form) always form  
$$s_e = s_0 + vT \sqrt{1 - \left(\frac{v}{v_0}\right)^8}$$
 maintain steady state gap. in jam also

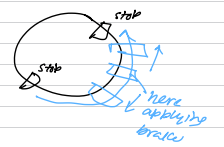
Now → Density ↓ ~ steady state (Free traffic steady state)

Then → Density ↑ ~ steady state (Congested steady state)

Steady state always achieved.

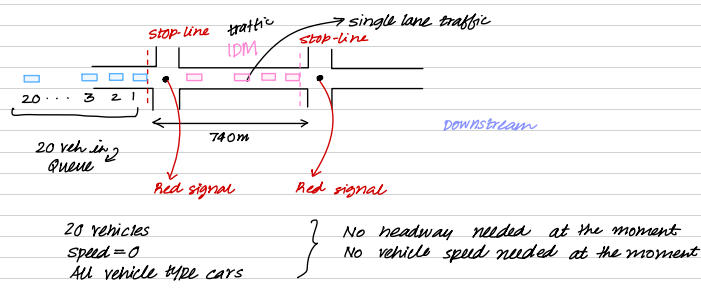


Now → Making two traffic lights  
 $b = 0.9 \text{ m/s}^2$



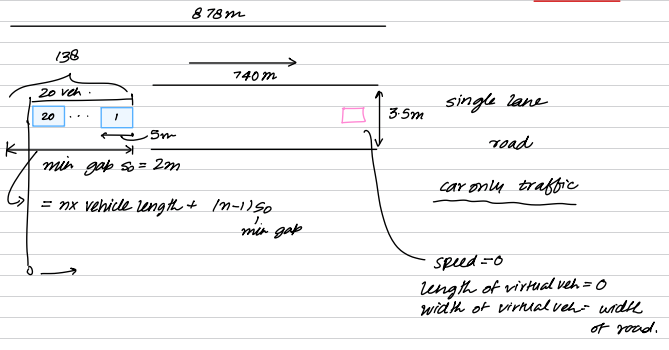
b ↑ then apply brake 's' distance  $v_0$ .  
b → (Anticipation Parameter)

CITY TRAFFIC



Assume: Downward traffic signal always red.

MOBIL (A lane changing model)  
Discrete lane changing model  
lab # 3



IDM Model

$$\frac{dv}{dt} = a \left( 1 - \left(\frac{v}{v_0}\right)^8 - \left(\frac{s_1}{s}\right)^2 \right)$$

$$= s_0 + vT + \frac{v \Delta v}{2\sqrt{ab}}$$

$$= s_0 + \max \left( 0, vT + \frac{v \Delta v}{2\sqrt{ab}} \right)$$

always maintain a min. ↓  
 $= s_0 + (-v)$

Virtual veh. speed = 0  
length of virtual veh = 0  
width of virtual veh = length of stop line. } } }  
veh. won't stop later.

$$\Delta v = v_f - v_f$$

$$\Delta v = 0$$

$$v_f = 0$$

$$\Delta v = v(\text{counter}) - v(\text{counter} - 1)$$

$$s_2 = \text{vehicle}(\text{counter} - 1) - \text{length of vehicle} - \text{vehicle}(\text{counter})$$

time step.  
 $\Delta t = 0.1 \text{ sec}$   
 $\Delta t = 1 \text{ sec}$   
 $t = 0.2 \text{ sec}$   
 $\Delta t \leq \text{Average Driver Reaction Time}$   
0.8 sec.

Why?  
But driver react in this time

$$\Delta t = 2 \text{ sec}$$

$$vT + \frac{v \Delta v}{2\sqrt{ab}}$$

$$s' = s_0 + \max \left( 0, vT, \frac{v \Delta v}{2\sqrt{ab}} \right)$$

Separately - why?

$t = 1 \text{ sec}$  time step = 1 sec  
 First veh. second veh.  
 ACC  $\Delta v = v(1 \text{ sec}) - v(0 \text{ sec})$   
 update speed }  $t = 2 \text{ sec}$   
 update position }  
 $v = u + at$   
 $v = u + at$   
 second veh.  
 $\Delta v = v(1 \text{ sec}) - v(0 \text{ sec})$   
 $s = \text{Leader - veh. position}$

Apply act at starting time (IDM Model)

$t = 0 \quad \Delta t = 0.1 \text{ sec}$

$t = 140 \text{ sec}$

$v = 0 \text{ km} \rightarrow \text{IDM model ACC (20)} \rightarrow \text{Update speed, position}$

$t = 0.1 \text{ sec} \rightarrow$

$t = 140 \text{ sec} = \frac{140}{0.01} = 14000$

IDM Model

31 Jan (Wed)

- Simulation Software
- Calibrate simulation model using real world data.
- Find optimum parameter values
- Replicate different driving regimes closely with field data.

Types of data in traffic flow

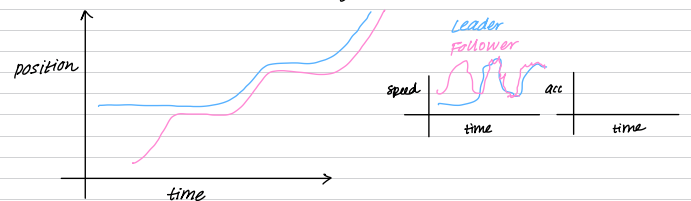
1. cross-sectional data ( $Q, P, V$ )
  2. trajectory (spatio-temporal dynamics of all vehicles)
- XPCD (x-floating car data) → spatio-temporal dynamics (vehicles)

Trajectory Data → Better for calibration  
 Two trajectories — NGSIM (New Generation Sim.)  
 open-source data  
 US DOT collected this data

F L  
 Follower Leader

position → primary data - using this data  
 speed } derived quantity  
 Acc }

Calc. speed & acc using primary data



Why choose this trajectory data?

- 1) Follower — free accelerating ( $t < 340 \text{ s}$ )
- 2) Cruising at desired speed ( $340 \text{ s} < t < 342 \text{ s}$ )

3) approaching a standing vehicle from large distance ( $342 \text{ s} \leq t \leq 350 \text{ s}$ ) → (b) comfortable deceleration

4) accelerating behind leader ( $350 \text{ s} \leq t < 360 \text{ s}$ )

5) Following the leader near steady state ( $360 \text{ s} \leq t \leq 365 \text{ s}$ )  
 ↳ (T) desired time headway

6) decelerating behind a leader ( $365 < t < 375 \text{ s}$ )

7) standing ( $t > 375 \text{ s}$ ) →  $s_0$  (min. gap)

position → primary data (NGSIM)

$t_i \quad x_i$   
 $t_{i+1} \quad x_{i+1}$   
 $t_{i+2} \quad x_{i+2}$   
 (point data)

speed: → Numerical scheme (Finite Difference Method)

Taylor Series  
 $f(t+\Delta t) = f(t) + \frac{\Delta t}{1!} f'(t) + \frac{\Delta t^2}{2!} f''(t) + \dots$   
 $f(t-\Delta t) = f(t) - \frac{\Delta t}{1!} f'(t) + \frac{\Delta t^2}{2!} f''(t) - \dots$   
 $t < \xi < t + \Delta t$  (MVT)

$$\frac{f(t+\Delta t) - f(t)}{\Delta t} = f'(t) + \frac{\Delta t}{2!} f''(\xi)$$

$$\frac{f(t+\Delta t) - f(t) - f'(t)\Delta t}{\Delta t} = \frac{\Delta t}{2!} f''(\xi)$$

Error / Remainder term

Order of accuracy =  $O(\Delta t)$

F.D.  $f'(t) = \frac{f(t+\Delta t) - f(t)}{\Delta t} \rightsquigarrow O(\Delta t)$  accuracy order

B.D.  $f'(t) = \frac{f(t) - f(t-\Delta t)}{\Delta t} \rightsquigarrow O(\Delta t)$  accuracy order

C.D.  $f(t+\Delta t) = f(t) + \frac{\Delta t}{1} f'(t) + \frac{\Delta t^2}{2!} f''(t) + \frac{\Delta t^3}{3!} f'''(\xi_1)$   
 $f(t-\Delta t) = f(t) - \frac{\Delta t}{1} f'(t) + \frac{\Delta t^2}{2!} f''(t) - \frac{\Delta t^3}{3!} f'''(\xi_2)$

$$\frac{f(t+\Delta t) - f(t-\Delta t)}{\Delta t} = 2f'(t) + \frac{\Delta t^2}{3!} [f'''(\xi_1) + f'''(\xi_2)]$$

$$\frac{f(t+\Delta t) - f(t-\Delta t)}{2\Delta t} - f'(t) = \frac{\Delta t^2}{12} [f'''(\xi_1) + f'''(\xi_2)]$$

order of accuracy:  $O(\Delta t^2)$

for  $\Delta t < 1$ , error is reduced (lesser error)  
 that's why we will use central difference.

✓  $f'(t) = \frac{f(t+\Delta t) - f(t-\Delta t)}{2\Delta t} = \text{speed}$

$f''(t) = \frac{f'(t+\Delta t) - f'(t-\Delta t)}{2\Delta t} \Rightarrow O(\Delta t^2)$

✓  $f''(t) = \frac{f(t+\Delta t) - 2f(t) + f(t-\Delta t)}{(\Delta t)^2} \Rightarrow O(\Delta t^4)$  Better Accuracy  
 ↳ = acceleration

We taking  $\Delta t = 0.1 \text{ sec}$ ,  $f'(t) = 10$  times amplified the error  
 $f''(t) = 100$  times amplified (huge)  
 $f'''(t) = \dots$  primary quantity  
 $\dots$  stick to it.

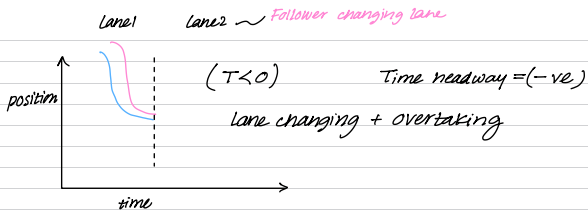
$v = -v$  } can't go -ve

De-Noise

Inconsistencies in NGSIM data

- 1) Negative gap
- 2) Negative speed
- 3) Unreasonable value of acceleration  $a = \pm 60 \text{ m/s}^2$
- 4) sudden jumps vehicle forward and backwards

Negative gap — collision or overtake



Internal consistency NGSIM — Position, speed, acc.

$$v = \frac{dx}{dt} \quad \dot{v} = \frac{dv}{dt}$$

$$x(t) = x(0) + \int_0^t v(t') dt' \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{can't apply on discrete case.}$$

$$x(t) = x(0) + \sum_0^n v(t') \Delta t$$

$$v(t) = v(0) + \int_0^t a(t') dt$$

$$v(t) = v(0) + \sum_0^n a(t') \Delta t, \quad \tau = \frac{t}{\Delta t}$$

lab #4

Internal consistency

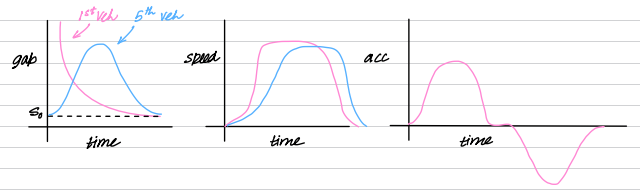
Platoon consistency

Initial gap ( $s_0$ ) also consistent

$$s(0) = x_f(0) - L - x_l(0)$$

$$s(t) = s_0 + \int_0^t (v_l - v_f) dt \quad \forall t \text{ (all time)}$$

If you consider position → Central Difference Method.  
 Automatically platoon consistency satisfied.



ref time = 0, 0.1, 0.2

time stat = [ ]

time stat = [ 0, 0.1, ..., 1.2 ]

gap stat, ...

time stat = [ 0.5, ..., 5 ]

$s, l, \dots$

speed slot

speed slot = [ - - - - ]

magnified

[ - - - - ] 0.1  
 [ - - - - ] 0.2

[ - - - - ]

100 sec

$\Delta t = 1 \text{ sec}$

time stat 101x1

gap stat 101x5

speed stat 101x5

acc stat 101x5

10M Model — Discontinuous graph — lane changing  
 (Active & Passive lane changing)



$$\text{error} = (x_s)_F - (x_d)_F \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Later}$$

De-Noise

Convolution ~ combining two functions to produce the new function.  
 $(f * g) \rightarrow (f * g)$

$$(f * g) = \int_{-\infty}^{\infty} g(t) f(t - \tau) d\tau$$

our data is discrete — we discrete convolution technique

$$(f * g) = \sum_{-\infty}^{+\infty} g(t) f(t - \tau)$$

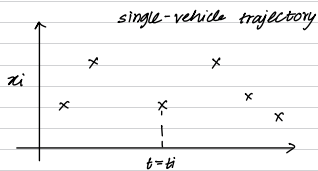
vehicle position  
 (from field data)

We have to construct this function

How to construct this function?  
 we have to construct this  $\phi$  mathematically.

New Lec

De-Noise or  
 Data smoothing technique



Discrete convolution

$$(f * g) = \sum_{-\infty}^{\infty} f(t) g(t - \tau)$$

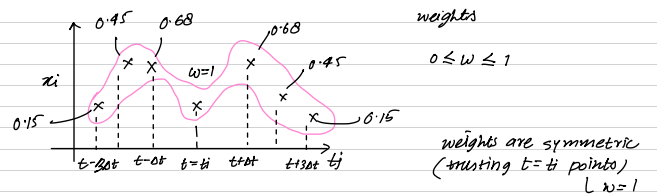
$f(t) \rightarrow$  from the data

weight function  $g(t - \tau)$

Kernel function

$x_{t=ti} \rightarrow$  smooth partition (or) position (after removing the noise)

traffic is non-linear.



$\phi(t - \tau)$  Kernel function

$\phi \rightarrow$  localised (upto some local range)

$|t - \tau| \rightarrow \infty$   
 $\phi \rightarrow 0$  } that is a localised function

$$\tau = t_i$$

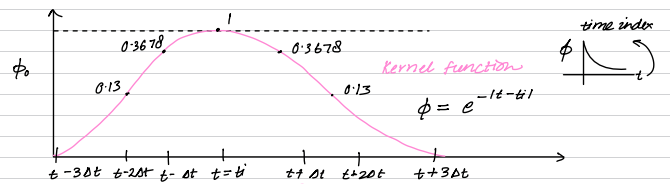
$$t = (t - 3\Delta t, t + 3\Delta t)$$

$\phi \rightarrow 0$   $|t - 3\Delta t|$   $|t + 3\Delta t|$

$\phi_0 \rightarrow$  maximum at  $\phi(t - \tau) = 1$   
 $t - \tau = 0$  or  $t = t_i, \tau = t_i$   
 $t_i - t_i = 0$

$\phi_0 \rightarrow$  continuous function of  $|t - \tau|$   
 $\phi_0 \rightarrow$  monotonically decreasing function  
 $\phi_0 \rightarrow$  symmetric

If you apply any filter in data, then there will be artifacts.  
 errors in the data. data is correct but wrong filter choice.



$\tilde{x} = \phi_0 x$   
 Kernel  
 position  
 convoluted function  
 $\phi = e^{-|t-t_i|}$   
 $\phi = e^{-0} = 1$  ( $t = t_i$ )  
 $\phi = e^{-|1-4|} = e^{-3} = 0.050$  ( $k=i$ )  
 $\phi = e^{-|3-4|} = e^{-1} = 0.3678$   
 $\phi = e^{-|2-4|} = e^{-2} = 0.13$

Data - equidistant in time  
 instead of going to time, you have to use time index.  
 ( $\Delta t = 0.1 \text{ sec}$ )  
 $t \rightarrow$  time index =  $k$   
 $\tilde{t} \rightarrow$  time index =  $i$

$\phi = e^{-\frac{|k-i|}{T}}$   
 $T$  higher - green curve  
 $T$  lower - pink curve  
 exponential moving average method.

$T$  - controls the shape of the curve  
 $T$  higher - green curve  
 $T$  lower - pink curve

$\tilde{x}_\alpha(i) = \sum_{k=i-D}^{i+D} x_\alpha(k) e^{-\frac{|i-k|}{\Delta}}$   
 kernel function  
 field data  
 position  
 $i=4, D=3$

Now, normalise  $\tilde{x}_\alpha(i)$   
 $\tilde{x}_\alpha(i) = \frac{1}{\sum_{k=i-D}^{i+D} e^{-\frac{|i-k|}{\Delta}}} \sum_{k=i-D}^{i+D} x_\alpha(k) e^{-\frac{|i-k|}{\Delta}}$  { Normalization }  
 $\frac{|i-k|}{\Delta} < \Delta$   
 always  
 low pass filter

low-pass filter ✓  
 high-pass filter

Install MATLAB  
 Full Chazk

7 Feb 2024, Wed.  
 Data smoothing  
 $\phi(t) = \sum \exp\left(-\frac{|t-t_i|}{\tau}\right)$  a model parameter  
 $\tilde{x}(t_i) = \sum \phi_0(t-t_i) x(t)$  reference of time  
 $\tilde{x}(t_i) = \frac{1}{\sum \phi_0(t-t_i)} \sum \phi_0(t-t_i) x(t)$  time index equidistant in time  
 $\tilde{x}(i) = \frac{1}{\sum_{k=i-D}^{i+D} \phi_0(k-i)} \sum_{k=i-D}^{i+D} \exp\left(-\frac{|k-i|}{\Delta}\right) x(k)$

$\Delta = \frac{\tau}{dt}$   $dt =$  time interval b/w two points  
 $T = 10 \text{ sec}$   
 $dt = 2 \text{ sec}$   
 $\alpha = 3$   
 $D = \min(3 \times 5) = 15$

$\Delta = 5 \text{ points}$   
 $D = \min\{\alpha \Delta, i-1, N-i\}$   
 starting point  
 ending point  
 parameter

$i = 16$   
 $i = 16$   
 total no. of points are always odd in number.

Calibration + Data  
 Compare between measured and simulated quantity.  
 Real world data vs car following model  
 Minimize difference b/w measured and simulated quantity  
 Find out optimum model parameters  
 Before calibration, simulation setup

smoothed position  
 L speed  
 L Acc  
 both leader and follower  
 1. internal consistency } already satisfied  
 2. platoon consistency }

Simulation setup  
 follower:  $x(0), v(0), a(0)$   
 leader:  $x_L(0), v_L(0)$   
 $t=0$   
 $t=t+\Delta t$   
 $t=t+2\Delta t$   
 external input  
 $\{x_L, v_L\}$   
 always this data comes from real world field data

$x^{sim}(0) = x^{data}(0)$   
 $v^{sim}(0) = v^{data}(0)$   
 $s^{sim}(0) = x_L^{data}(0) - L_L - x^{sim}(0)$   
 Apply IDM Model at time  $t=0$

ACC-IDM-model ( $s^{sim}, v^{sim}(0), \Delta v^{sim}$ )  
 $\left(\frac{dv}{dt}\right)_{t=0} = \Delta v^{sim}(0) = v^{sim}(0) - v_L^{data}(0)$   
 $s(1) = x_L^{data}(1) - L - x^{sim}(1)$   $\Delta t = 1 \text{ sec}$   
 $v = u + at$   
 $x_0 = s_0 + \frac{(v_L + v)}{2} \Delta t$   
 $x^{sim}(2) = v^{sim}(1) - v_L^{data}$   
 $v^{sim}(2)$

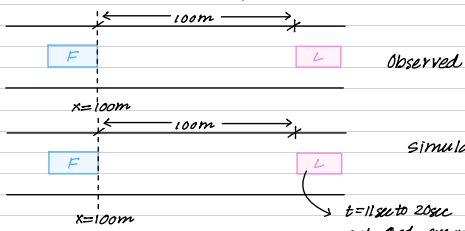


Parameters  $x, v, a, s$

Position  $x$

Obj. Func.  $\min \sum_{i=0}^n [x_i^{data} - x_i^{sim}]^2$

$\begin{Bmatrix} a \\ v_0 \\ T \\ s_0 \\ b \end{Bmatrix}$



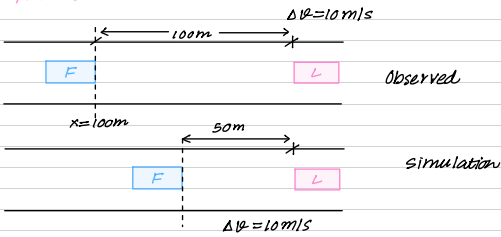
Observed

Simulation

$b = 11 \mu s$  to  $20 \mu s$   $\uparrow$  Acc gap red every time

zero error can't find parameters optimally can't find.

Speed  $v$



Observed

Simulation

$\Delta v \rightarrow$  correct parameter

error =  $\frac{1}{n} \sum (\Delta v^{data} - \Delta v^{sim})^2 = 0$

gap  $s$

$s = x_L - L - x_f$   
 $s = \Delta v$  } good variable

Kyun?

error =  $\frac{1}{n} \sum_{i=1}^n (s^{sim} - s^{data})^2$  minimize

absolute error =  $\frac{\frac{1}{n} \sum_{i=1}^n (s^{sim} - s^{data})^2}{\frac{1}{n} \sum_{i=1}^n (s^{data})^2}$

$s^{sim} = 110$   
 $s^{data} = 10$  }  $(s^{sim} - s^{data})^2 = 100^2 \sim$  numerator blows up  
 more sensitive to larger gap difference  
 free flow condition

relative error =  $\frac{1}{n} \sum_{i=1}^n \left( \frac{s^{sim} - s^{data}}{s^{data}} \right)^2$  more sensitive to small gap

$s^{sim} = 10m$   
 $s^{data} = 5m$  } error =  $\frac{10-5}{5} = 100\%$

$s^{sim} = 105m$   
 $s^{data} = 100m$  } error =  $\frac{105-100}{100} = 5\%$

for same diff. in  $s$ , different errors are coming up.

mix both

error mix =  $\frac{\sum_{i=1}^n (s^{sim} - s^{data})^2}{\sum_{i=1}^n |s^{data}|}$

Free traffic and congested traffic

{ analytical method }  
 { heuristics method }

(12 Feb, Tues)

Model

Data  $\rightarrow$  De-noise  
 Objective Function

$f = \text{Error}_{rel} = \frac{1}{n} \sum \left( \frac{s^{obs} - s^{sim}}{s^{obs}} \right)^2$

Objective Function  $\rightarrow$  minimize

$f(v_0, T, s_0, a, b)$  optimum model parameters

$v_0, T, s_0, a, b$

0 m/s (estimating optimum model parameters)

minimization problem

$f$  is continuous function

$f$  is not continuous function  $\rightarrow$  mostly happen (discontinuities)

lane changing happens, discontinuity happens!

evolutionary algorithm (Genetic Algorithms)



DNA's ATCG  $\rightarrow$  optimal or sub-optimal solution

General structure (GA)

1) Initialization

Assume Initial population (randomly)

$\begin{Bmatrix} v_0 = 15 \\ T = 1 \\ a = 1 \\ b = 1.5 \\ s_0 = 2.5 \end{Bmatrix}$  fitness function =  $\frac{1}{n} \sum \left( \frac{s^{obs} - s^{sim}}{s^{obs}} \right)^2$

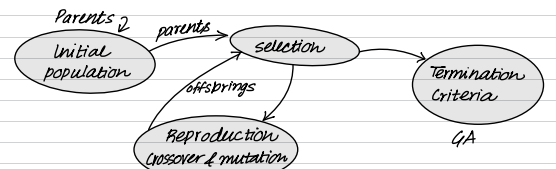
$\begin{Bmatrix} v_0 \\ T \\ a \\ b \\ s_0 \end{Bmatrix}$  200 populations Random values

{ LB } GA automatically generate random values b/w LB and UB for all parameters (200 fitness functions)

function  $f(x)$

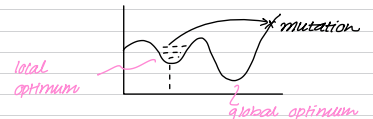
$\min f(x) \rightarrow x = v_0, T, a, b, s_0$

2) Evolution Loop



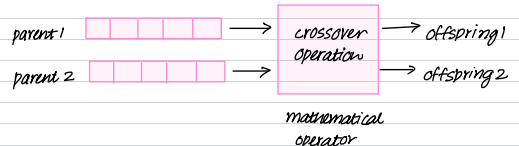
GA

- 1) Initialization (Initial population  $\rightarrow$  randomly)
- 2) Select parents & crossover
- 3) mutation  $\rightarrow$  chromosomes error in genes - increase explorative ability of GA - create new innovative solution

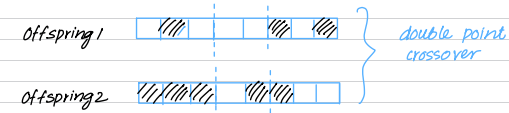
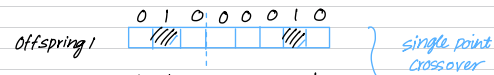
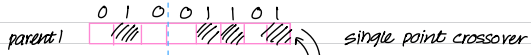


- 4) merge with population and offspring
- 5) Evaluate, sort and select
- 6) Check for termination criteria if not, go to step 2.

Crossover



mathematical operator



uniform crossover

$$x_1 = \{x_{11}, x_{12}, x_{13}, \dots, x_{1n}\}$$

$$x_2 = \{x_{21}, x_{22}, x_{23}, \dots, x_{2n}\}$$

$$a_2 = \{a_{21}, a_{22}, a_{23}, \dots, a_{2n}\}$$

$$a = \{a_1, a_2, a_3, \dots, a_n\}$$

$$a_i = 0 \text{ or } 1$$

$$y_1 = \{y_{11}, y_{12}, y_{13}, \dots, y_{1n}\}$$

$$y_2 = \{y_{21}, y_{22}, y_{23}, \dots, y_{2n}\}$$

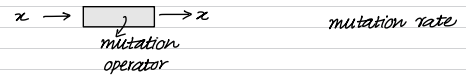
$$y_{1i} = a_i x_{1i} + (1 - a_i) x_{2i}$$

$$y_{2i} = (1 - a_i) x_{1i} + a_i x_{2i}$$

200 population  $\rightarrow$  200 offsprings

Mutation

- $\rightarrow$  error in the genes
  - $\rightarrow$  increase explorative ability of GA
  - $\rightarrow$  new solution (local optimum)
- two genes



$$x = (x_1, x_2, \dots, x_n) \quad x'_i = (x'_1, x'_2, \dots, x'_n)$$

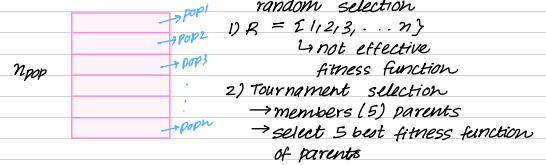
$$x_i \in \{0, 1\}$$

$$j = 1 \text{ to } n \text{ (RN)}$$

$$i = 10$$

$$x'_i = \begin{cases} x_i & j \neq i \\ 1 - x_i & i = j \end{cases}$$

parent selection

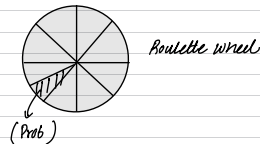


This is deterministic.

3) Roulette wheel selection

$$P_i = \frac{f_i}{\sum_{i=1}^n f_i} \quad (\text{Assign probability})$$

here  $n=10$



Casino ~ Gangster

MIT Professor

Real Life movie.

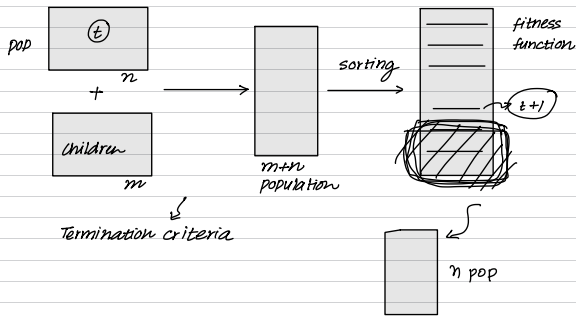
Italy

German

French

Italian

Merge with Parent and Offspring



14 Feb 2024, Wed

min  $f(V_0, T, a, b, S_0)$  discrete

$V_0 = LB \sim UB$  crossover operations

$T = LB \sim UB$

$a = LB \sim UB$

rel error =  $f = \frac{1}{n} \sum_{i=1}^n \left( \frac{S_{data} - S_{sim}}{S_{data}} \right)^2$

also Subs (same)

parent 1  $\rightarrow x_1 = (x_{11}, x_{12}, x_{13}, \dots, x_{1n})$  cross over

parent 2  $\rightarrow x_2 = (x_{21}, x_{22}, x_{23}, \dots, x_{2n})$

offspring 1  $\rightarrow y_1 = (y_{11}, y_{12}, y_{13}, \dots, y_{1n})$

offspring 2  $\rightarrow y_2 = (y_{21}, y_{22}, y_{23}, \dots, y_{2n})$

$a = \{a_1, a_2, a_3, \dots, a_n\}$   $a \in (0, 1)$  Random

$y_{1i} = a_i x_{1i} + (1 - a_i) x_{2i}$

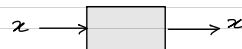
$y_{2i} = (1 - a_i) x_{1i} + a_i x_{2i}$

Crossover

$\alpha \in (-\delta, 1 + \delta)$   $\delta \rightarrow$  very very small number

It improves exploration ability of GA.

mutation



$(x_1, x_2, \dots, x_j, \dots, x_n) \quad (x'_1, x'_2, \dots, x'_j, \dots, x'_n)$

$x'_j = x_j + \delta$   $\delta \rightarrow$  follows uniform distribution  $(-\epsilon, +\epsilon)$  limited range

Better use Gaussian

$\delta \rightarrow$  Gaussian  $\sim N(0, \sigma^2)$

Most of the fall near mean (here mean = 0)

Revisit

min  $f = \frac{1}{n} \sum_{i=1}^n \left( \frac{S_{data} - S_{sim}}{S_{data}} \right)^2$

$S_{sim} = g(V_0, T, a, b, S_0)$

1) Initialization  $\rightarrow$  Initial population = 200

	set 1	set 2	...	set 200	
$V_0 \sim$	15	30			$V_0 \rightarrow 200$
$T \sim$	1	1.6			$T \rightarrow 200$
$a \sim$	0.1	4.0			$a \rightarrow 200$
$b \sim$	3.6	0.01			$b \rightarrow 200$
$S_0 \sim$	6	5			$S_0 \rightarrow 200$
	$\Rightarrow f_1$	$\Rightarrow f_2$		$\Rightarrow f_{200}$	

values

$f_1 =$  fitness function 1

$f_2 =$  fitness function 2

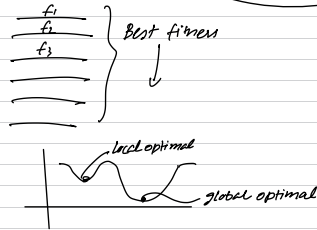
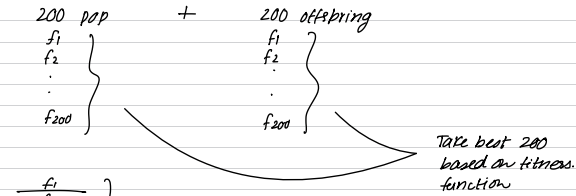
$\vdots$

$f_{200} =$  fitness function 200

crossover	$v_0$	$T$	$a$	$b$	$s_0$
pop1	15	1	0.1	3.6	6
pop2	30	1.6	4.0	0.01	5

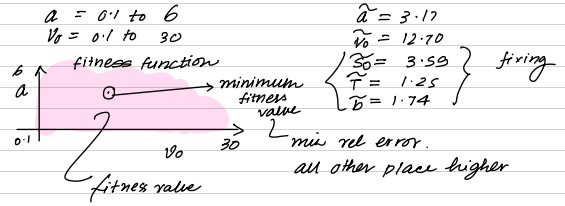
Similarly 200 population  
↓  
200 offsprings.

	$v_0$	$T$	$a$	$b$	$s_0$
offspring1	15	1	4	0.01	5
offspring2	30	1.6	0.1	3.6	5



leader raw data ~ After smoothing.  
CS-rel ~ cumulative rel. error  
global for ~ can be accessed anywhere  
... → continue

selection fare ~ select 2 pop out of 200 for crossover  
Max Generation ~ max 420 iterations  
↓  
max no. of iterations.



Fix any 3 and vary only 2 parameter.  
Plot contour plots.

optimum value around blue (min)  
← yellow (max).

16 Feb 2024, Thurs

$$\bar{a} = f(s, v, \Delta v)$$

leader acc ~ no acc required

not included in parameters.

At  $t=0$ ,

$$s_{sim} = x_L - L - X$$

$$s_{oss} = x_L - L - X$$

$s_1$   $s_2$   $s_3$   $s_4$   $s_5$   
↓ ↓ ↓ ↓ ↓  
param =  $(a, v_0, T, s_0, b)$

$$b_{002} x^{sim}(0) = x^{data}(0)$$

leader - externally controlled - field data

$t=0$  to 521 timesteps (521 data points)

$$\Delta v = v - v_0 \text{ \{ relative speed in IDM Model \}}$$

$$\frac{dvm}{t_{sum}} = \frac{v \Delta v}{2 \Delta t}$$

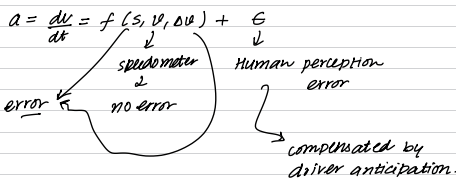
$$\frac{dvm}{t_{sum}} = \frac{\text{follower speed} \times \text{rel speed}}{2 \sqrt{\text{param}(1) \times \text{param}(5)}}$$

$$a = \frac{dv}{dt} = f(s, v, \Delta v)$$

$$v = u + at$$

$$s = s_0 + ut + \frac{1}{2} at^2$$

$t=0.1$  {  $x_L^{data}(1)$   $x_L^{sim}(1)$  } similarly repeat upto 521  
          {  $x_L^{data}(1)$   $v^{sim}(1)$  }



(Ya iss class K baad??)

always declare any data ~ in main function.  
Don't read anything inside iteration.

Genetic Algorithm ~ won't ask in Exam

16 Feb 2023

Not all model parameters

→ estimated

→ time 350s to 396s

$\{T, a, b, s_0, v_0\}$

→ time  $t > 386s$

gap decreasing

data 386s to 397s

desired time gap

$$T = (-ve)$$

Consider whole trajectory data

T → some error → certain errors

$v_0$  → desired speed  
 $a$  → max acceleration

can't find

anticipation behaviour

$$s = s_0 + vT + \frac{v \Delta v}{2 \Delta t}$$

$$v(\uparrow) \quad s(\downarrow)$$

$$T = \frac{s}{v}$$

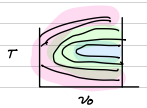
~ Not consistent with model

driver want to lane change.  
(anticipation behaviour).



Investigation of the parameter space

indifferent w.r.t. desired speed



IDM Model

→ Radar Data set (Germany)

data set does not contain

- freeflow
- approaching

desired speed

$v_0 = 20,000 \text{ km}$

↳ unconstrained optimization

You have to choose correct model.

The model parameter have to be orthogonal

Parameter Orthogonal

diff. driving regimes → situation represent by diff. parameters.

free acceleration →  $a$

crusing ( $v_0$ ) →  $v_0$

approaching →  $b$  comf. dec.

following →  $T$

standing →  $s_0$

fixed published values  $f_{max}(v_i)$

max of all stand in data

Every parameter represent one driving regime only.  
That is called parameter orthogonal.

Each parameter diff. parameter model | situation.

↳ if not, more than one then some correlation will be there.

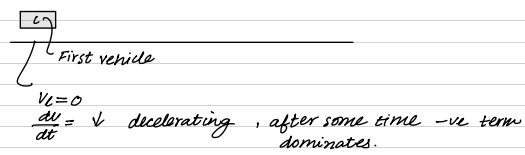
Ideal case

Each parameter  $\rightarrow$  only one driving scenario

$\hookrightarrow$  we can fix the problem even with incomplete database.  
1. Fix values from published values  
2. Fix values the max ( $v_i$ )  $\sim$  all  $v_i$  from data.

Quiz-1  $\frac{dv}{dt} = \frac{v_0(s) - v^2}{\tau} - \alpha(v - v_e)$   
Flexing  $\hookrightarrow$  not complete model linear term in rel. speed.

Model Completeness



whenever calibrating  $\sim$

1. Tipping point  $\rightarrow$  don't use this data  $\sim$  traffic jam
2. Test the data  $\rightarrow$  contain all driving regimes
3. Fix that particular parameter  
Other parameter  $\rightarrow$  calibrate  
data  
or  
published values
4. LB and UB  $\sim$  restrict
5. good initial guess
6. Plot fitness landscape  $\sim$  unimodal  
 $\hookrightarrow$  data completeness.

GA  $\sim$  algorithm good  $\sim$  also lot of others.

$\hookrightarrow$  optimization comes in this part.  
(Evolutionary Algorithms)

7. Robustness  $\sim$  Model must be robust.